This exam is due on paper at the start of class on Monday. The exam must be completed within a single 180-minute (three-hour) block of time. Your block begins when you open (or peek inside) this packet.

The exam is open-note: You may use your class notes, your old homework, your textbook, and our winter 2026 Math 354 course web site. You may not share any of these materials with other students. You may not discuss the exam with anyone but me until Monday at the start of class. You may not consult other books, Internet sites, artificial intelligences, mathematical software, etc.

I will try to check my e-mail frequently during Saturday evening and Sunday evening. At other times of the exam period, I might be busy with other tasks, but I will check as often as I can. Feel free to ask clarifying questions. If you cannot obtain clarification on a problem, then explain your interpretation of the problem in your solution, so that I can judge your solution relative to your interpretation. You might lose points, if your interpretation makes a problem drastically easier than it should be. Certainly you should never interpret a problem in a way that renders it trivial.

You may type or write your solutions. Your solutions should be thorough, self-explanatory, neat, concise, and polished. Always show enough work and justification so that a typical classmate could understand your solutions. You may cite material (definitions, theorems, examples, etc.) from class, homework, and assigned textbook readings up to and including Day 18. You do not have to redevelop or reprove that material. If you wish to use other material, then you have to develop it. Correct answers without supporting work rarely earn full credit.

If you cannot solve a problem, then write a brief summary of what you know that is relevant, and the approaches that you've tried. Partial credit is often awarded.

Good luck. :)

This page's text has nothing to do with the exam. It is placed here merely to take up space. You are not obligated to read it at all. Actually, it is part of a restaurant menu backwards.

? .snaeb dna ecir ,dalas ecuttel htiw devres ,telif hsif dedaerB odacsep ed eteliF .71

94.01 .srekcarc fo ediS .odacova dna ollag ed ocip ,ecuas liatkcoc htiw devres liatkcoc pmirhS senoramac ed liatkcoC .61

94.21 .snaeb dna ,ecir ,dalas ecuttel htiw devres pmirhs obmuj dedaerB sodazinapme senoramaC .51

94.01 .snaeb dna ,ecir ,dalas ecuttel htiw devres ,deirf-peed hsif aipalit elohW sarrajoM .41

94.01 .eseehc dna maerc ruos ,ecuttel ,snoino ,ecuas ,)nekcihc ro ,krop ,kaets feeb(taem fo eciohc ruoy htiw deppot seittap nroc kciht edamdnaH)ozirohc o agnit ,rotsap ,adasa ,anicec(sepoS .31

hcae 99.2 doofaeS/socsiraM ,sepoS .maerc ruos dna ecuas der ro neerg ,ollag ed ocip ,ecuttel htiw devreS .)krop ro kaets feeb ,nekcihc(taem fo eciohc ruoy htiw allidaseuQ)ozirohc o rotsap ,adasa ,agnit(allidaseuQ .21

99.8 .ecuttel htiw devres allidaseuq eseehC oseuq ed allidaseuQ .11

99.6 .ecuas eltopihc ni nekcihc ro ,egasuas nacixeM ,krop ,kaets htiw ocat nroC)agnit o ozirohc ,rotsap ,adasa(socaT .01

hcae 52.2 .mah ro eltopihc htiw nekcihc dedderhs ,krop ,kaets feeb ,nekcihc dedaerb fo eciohc htiw hciwdnas nacixeM lanoitidarT)nomaj o agnit ,rotsap ,adasa ,asenalim(atroT .9

99.7 .maerc ruos dna ,ollag ed ocip ,odacova ,dalas ,snaeb ,ecir htiw devreS .)krop(satinrac ro ,nekcihc ,feeb aocabrab :snoitpo taeM ollitalP .8

99.9 .maerc ruos dna ecuas ,dalas ecuttel htiw devreS .)egasuas nacixeM ro krop ,nekcihc ,feeb(taem fo eciohc ruoy dna eseehc ,snaeb ,ecir htiw dellif allitrot ruolF)ozirohc o rotsap ,agnit ,adasa(otirruB .7

99.7 allidaseuQ ,socaT ,satroT ,sotirruB .maerc ruos dna ollag ed ocip ,odacova ,dalas ecuttel htiw devres ,kaets triks feeb dellirG arehcarrA .6

99.9 .edis no snaeb dna eciR .sutcac dellirg dna snoino ,sreppep onepalaj ,eseehc htiw deppot ,egasuas nacixeM dna ,anicec ,sbir feeb dellirG 2 arap sadasA senraC .5

?99.02 .ollag ed ocip dna ,maerc ruos ,dalas ecuttel htiw devreS .snoino htiw deppot kaets feeb dellirG odallobecnE cetsiB .4

99.9 .maerc ruos dna odacova ,dalas ecuttel htiw devres ,kaets dedaerB ocreuP o olloP ed asenaliM .3

94.9 .ollag ed ocip dna maerc ruos ,dalas ecuttel htiw devreS .snoino dna sreppep onepalaj htiw deppot ,ecuas reppep ollijaug laiceps ruo htiw detaniram kaets krop dellirG adalihcnE enraC .2

99.9 .dalas ecuttel htiw devreS .snoino dna sreppep onepalaj htiw deppot ,kaets feeb niht detaniram dellirG aniceC .1

There are three problems labeled B–D, along with time checks A, E, which are not optional.

A. On what day, at what time, did you start this exam?

For Problem B, let $\mathbb{R}_{>0} = (0, \infty)$ be the space of positive real numbers. Let $p : \mathbb{R}_{>0} \times \mathbb{R} \to \mathbb{R}_{>0} \times \mathbb{S}^1$ be given by $p(r, t) = (r, \cos t, \sin t)$.

B.A. Prove that p is a covering map, from the definition of covering map (rather than citing the result or using any other strategy).

B.B. Can this covering map be used to calculate the fundamental group of $\mathbb{R}_{>0} \times \mathbb{S}^1$? If so, then carry out that calculation. If not, then explain precisely why not.

For Problem C, let X be a space such that the identity map $X \to X$ is homotopic to the constant map $X \to \{x_0\} \subseteq X$ for some $x_0 \in X$.

C.A. Let Y be another space. Prove that any two maps $f, g : Y \to X$ are homotopic.

C.B. Prove that X is path-connected.

For problem D, we're going to assume a concept from more advanced topology and investigate some of its consequences. The concept is the *multiplicity* of a map $h : \mathbb{S}^2 \to \mathbb{S}^2$. The multiplicity of h is an integer, $\text{mult}(h)$, with these properties:

1. The first coordinate reflection $h(x, y, z) = (-x, y, z)$ has multiplicity -1.
2. $\text{mult}(h \circ k) = \text{mult}(h) \cdot \text{mult}(k)$.
3. If $h \simeq k$, then $\text{mult}(h) = \text{mult}(k)$.

You may assume these properties. Also, it might be helpful to know these matrices:

$$A_t = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos t & -\sin t \\ 0 & \sin t & \cos t \end{bmatrix}, \quad B_t = \begin{bmatrix} \cos t & 0 & \sin t \\ 0 & 1 & 0 \\ -\sin t & 0 & \cos t \end{bmatrix}, \quad C_t = \begin{bmatrix} \cos t & -\sin t & 0 \\ \sin t & \cos t & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

The matrix A_t represents rotation of $\mathbb{R}^3$ about the x-axis through an angle of t radians (in a right-handed sense). Similarly, B_t and C_t are rotations of $\mathbb{R}^3$ about the y- and z-axes.

D.A. What is the multiplicity of the identity map $\mathbb{S}^2 \to \mathbb{S}^2$?

D.B. Show that the other two coordinate reflections — that is, $(x, y, z) \mapsto (x, -y, z)$ and $(x, y, z) \mapsto (x, y, -z)$ — also have multiplicity -1.

D.C. What is the multiplicity of the antipodal map $a(x, y, z) = -(x, y, z)$?

D.D. Suppose that $\text{mult}(h) \neq -1$. Prove that h has a fixed point — that is, there exists an $x \in \mathbb{S}^2$ such that $h(x) = x$.

E. On what day, at what time, are you finishing this exam?

Again, this page's text has nothing to do with the exam. If you ever become a pharmaceutical executive, there are some good drug names in here: sodazinapme, lanoitidart, detaniram, ...

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