Please write your name at the top of this page and nowhere else.

In addition to this cover page, there should be four problems, A–D, spread over four pages.

No notes, books, calculators, computers, etc. are allowed.

If a problem is unclear and you cannot obtain clarification, then write your interpretation of the problem, so that I can evaluate your solution relative to your interpretation. You might be penalized, if your interpretation makes the problem much easier than it should be. Certainly you should never interpret a problem in a way that renders it trivial.

All problems require explanation. Incorrect answers with good work often earn partial credit. Correct answers without justification rarely earn full credit. Write as if your audience is a typical classmate — not a professor. In doing so, you (hopefully) show enough detail, that I can evaluate whether you understand your arguments.

Except where otherwise directed, you may cite material (definitions, theorems, etc.) that we have defined or proved in class, in the assigned textbook readings, or in the assigned homework. You do not need to re-define or re-prove any of that material. You may not cite other material without developing it first. When in doubt, ask me.

Some problems ask for proofs. On a short exam such as this, you might not have time to make every proof rigorous. Try to hit all of the important concepts in your proof, even if it means leaving logical gaps. Mark each significant logical gap, so that I know that you know that it is a gap. For example, you might write "Claim: . . . ", finish the proof using the claim, and circle back to prove the claim only if you have time.

Pictures often help both you and your reader.

You have 70 minutes. Good luck. :)

We have learned that, for a surjection $f : X_T \to Y$, the quotient topology on Y is the *something*-est topology that does *something*. In this problem, we consider the opposite situation, where $f : X \to Y_S$ is an injection, and we want to make a topology T on X.

A.A. The topology T should be the *what*-est topology that does *what*?

A.B. Precisely describe the open sets of T. You might use the phrase "if and only if".

A.C. Of the constructions that we have studied, which one is most similar to this new one?

B. Let $f : [0, 1] \to [0, 1]$ be continuous. Prove that f has a fixed point — that is, there exists an x such that $f(x) = x$. (Hint: Draw a picture.)

Let $p : X \times Y \to X$ and $q : X \times Y \to Y$ be the projections. Suppose that X has a topology T and Y a topology S. Just before we defined the product topology, a brave student proposed that a subset U of $X \times Y$ should be declared to be open if both $p(U) \subseteq X_T$ and $q(U) \subseteq Y_S$ are open. This declaration defines a sub-basis, which induces some topology Q on $X \times Y$. Henceforth, focus on the case where $X_T = Y_S = \mathbb{R}$ with its standard topology.

C.A. How does Q compare to the product topology? Equal? Finer? Coarser? Not comparable?

C.B. From C.A, can you immediately conclude that p and q, as functions from $(X \times Y)_Q$ to X_T and Y_S respectively, are continuous? Or can you not? (Do not do a big new proof here.)

In this problem, you may *not* just cite the results from class or textbook; complete proofs are expected. Recall that a space X_T is T_1 if for all $x \neq y \in X_T$ there exist open sets $U, V \subseteq X_T$ such that $x \in U \not\ni y$ and $x \notin V \ni y$.

D.A. Assume that X_T is T_1. Prove that, for all $x \in X_T$, the one-point set $\{x\}$ is closed in X_T.

D.B. Conversely, assume that every one-point set $\{x\}$ is closed in X_T, and prove that X_T is T_1.