This is the last homework assignment that you will hand in. I hope that it's a bit fun.

Recall from class that the *Euler characteristic* $\chi = V - E + F$ can be defined for any "well-behaved" division of a compact surface into V vertices, E edges, and F faces. The notion of well-behavedness is made precise in Munkres's definition of a triangulation in Section 78. The only difference here is that we allow faces with more than three sides. Such faces can be subdivided into triangles to yield a triangulation in the sense of Munkres, without changing χ. So the difference is superficial.

A *Platonic solid* is a sphere $\mathbb{S}^2$ divided into vertices, edges, and faces in a well-behaved way, with two additional stipulations. First, there is an integer $D \geq 3$ such that exactly D edges meet at each vertex. Second, there is an integer $S \geq 3$ such that each face has exactly S sides. (If the word "solid" confuses you, what we're calling a Platonic solid is actually the boundary surface of a Platonic solid.) Platonic solids also have rigid geometric symmetries, but it turns out that you don't need any geometry to classify the Platonic solids. The classification is topologically determined, via the Euler characteristic.

A. List all possible Platonic solids, by enumerating all possible combinations of D and S. (This is a medium-length or possibly long exercise. If you get stuck, then consult the three-step solution sketch on the back of this sheet.)

A classic soccer ball (such as the Adidas Telstar — but don't look it up yet!) is made from H hexagons and P pentagons sewn together along their sides, such that exactly three faces meet at each vertex. Topologically, these hexagons and pentagons constitute a well-behaved division of a sphere.

B. Which values of P are possible? (This is a medium-length or possibly long exercise. If you get stuck, then consult the two-step solution sketch on the back of this sheet.)

In class, we derived an expression for the Euler characteristic $\chi(X \# Y)$ in terms of $\chi(X)$ and $\chi(Y)$, for compact, connected surfaces X and Y. We also asserted that our list of compact connected surfaces is exhaustive; there aren't any other ones. We'll outline that proof at the start of next class. For now, assume that it's true.

C.A. Compute $\chi(X)$ for every compact, connected surface X.

C.B. Check that your answer to Problem C.A is compatible with the claim that I made on the previous homework about a one-to-two exchange rate.

Here are three stepping stones to solving Problem A.

A.A. Prove that $VD = 2E = FS$.

A.B. Prove that $\frac{1}{D} + \frac{1}{S} > \frac{1}{2}$.

A.C. Based on Problem A.B, enumerate all possible combinations of D and S.

Here are two stepping stones to solving Problem B.

B.A. Prove that P must be a multiple of 2 and a multiple of 3.

B.B. Prove that $P = 12$ is the only possible value of P.