A. Let X be path connected and $x, y \in X$. Suppose that $\pi_1(X, x)$ is commutative. Prove that, for any two paths α, β from x to y, $\alpha_\# = \beta_\#$. (This is a medium-length exercise. It is half of Section 52 Exercise 3. You are invited, but not required, to do the other half.)

B. Section 52 Exercise 4. (This is a short exercise.)

C. Under the same hypotheses as Section 52 Exercise 4, let $i : A \hookrightarrow X$ be the inclusion. Prove that i_* is injective. (This is a short exercise.)

If the preceding exercises B and C seem too easy — like maybe you're missing something — then apply that unsettled feeling to this question: In general (not under the hypotheses of Section 52 Exercise 4), if $x \in Y \subseteq X$ and $i : Y \hookrightarrow X$ is the inclusion, then is $i_* : \pi_1(Y, x) \to \pi_1(X, x)$ injective? The answer is no, although we're not ready to prove so yet.