Some of these problems are about the material that we discussed on Day 07 of class. The homework is catching up with the classwork. That's okay.

A. Section 17 Exercise 12, which is about subspaces of Hausdorff spaces.

B. Fix integers $m \geq n \geq 0$. Let X be a subspace of $\mathbb{R}^m$ such that every $x \in X$ has a neighborhood that is homeomorphic to an open set in $\mathbb{R}^n$. Prove that X is an n-manifold. (Hint: You may cite Lemma 16.1.)

Epilogue: My understanding of history is that manifolds were originally viewed as subspaces of $\mathbb{R}^m$. Then mathematicians started preferring a more abstract, intrinsic treatment that didn't assume an embedding into $\mathbb{R}^m$. So the question naturally arose: Even if you don't assume an embedding, does an embedding exist anyway? This exercise shows that the existence of an embedding requires the Hausdorffness and countable basis axioms of n-manifolds. What's more surprising is the converse — that these assumptions are enough to guarantee an embedding. We might explore that theorem later.

Earlier in the course, we endowed the interval $[0, 1)$ with the quotient topology Q arising from $f : \mathbb{R} \to [0, 1)$ defined by $f(x) = x - \lfloor x \rfloor$. In homework we proved that $[0, 1)_Q$ is homeomorphic to the circle $\mathbb{S}^1$. Let S be the subspace topology on $[0, 1) \subseteq \mathbb{R}$.

C. Prove that $[0, 1)_Q$ is not homeomorphic to $[0, 1)_S$. (This exercise is short.)

D. Section 24 Exercise 2, which is about a function $f : \mathbb{S}^1 \to \mathbb{R}$.

Epilogue: This is the first really surprising result of the course, I think. It's a special case of the Borsuk-Ulam theorem, which is related to the Ham Sandwich theorem and other fun stuff that we'll encounter later. Exercise 3 is of a similar spirit (but I'm not asking you to hand in Exercise 3).

Also, Section 24 Exercise 9 is pretty enjoyable. It's not typical of how problems are posed or solved in topology, but a little variety can be refreshing, so give it a try if you like.