Let Y be a subset of a space X_T. Section 17 defines several closely related concepts: the *interior* Y°, the *closure* $\overline{Y}$, and the set Y' of *limit points*. Also, Exercise 19 defines the *boundary* ∂Y. Read these definitions. (The notations for the interior and boundary are different in my mathematical subculture than in Munkres's.) Here's an example. In $\mathbb{R}^n$, consider the ball $Y = B(\vec{0}, 1) = \{\vec{x} : |\vec{x}| < 1\}$ and the sphere $\mathbb{S}^{n-1} = \{\vec{x} : |\vec{x}| = 1\}$. Then

$$Y^\circ = Y, \quad \overline{Y} = Y' = \{\vec{x} : |\vec{x}| \leq 1\}, \quad \partial Y = \mathbb{S}^{n-1}.$$

A. In $\mathbb{R}^2$, let Y be the graph of $y = \sin(1/x)$ for $x > 0$. What are Y° and Y'? You don't need to rigorously prove your answers, but do be careful. Then, citing a theorem from Section 17, what is $\overline{Y}$?

B. In an arbitrary metric space (X, d), is it always true that $\overline{B(x, \epsilon)} = \{y : d(x, y) \leq \epsilon\}$? Prove so or give a counter-example.

C. Section 21 Exercise 2 (about embeddings, which are defined in Section 18).

D. Section 17 Exercise 13 (about the diagonal).

Want more optional practice? Try Section 17 Exercises 2, 7, 8, 9, 20, and 19. And Section 20 Exercise 3a.