A. Section 16 Exercise 4. (A side effect of this exercise is the definition of *open map*, which is fairly important.)

In class on Day 03, we used the surjection $f : \mathbb{R} \to [0, 1)$ defined by $f(x) = x - \lfloor x \rfloor$ to endow $[0, 1)$ with a non-standard topology T (the quotient topology induced by f). I argued intuitively that $[0, 1)_T$ is a circle. Let's make that more rigorous. I claim that $g : [0, 1)_T \to \mathbb{S}^1 \subseteq \mathbb{R}^2$ defined by $g(x) = (\cos 2\pi x, \sin 2\pi x)$ is a homeomorphism.

B. Taking it for granted that g is bijective onto $\mathbb{S}^1$, prove the rest of what you need, to conclude that g is a homeomorphism. (This task is not especially difficult, but your proof should explicitly address the fact that there are two kinds of open sets in $[0, 1)_T$: the ones that contain 0, and the ones that don't.)

Epilogue: In the preceding problem, my main goal is to get you to think about what constitutes a rigorous argument, and whether that argument is helpful to a human. All proofs written by humans have logical gaps, which the reader is expected to fill in. How big of gaps are allowed? That depends on the intended audience. (In this course, your intended audience is your fellow classmates, as the syllabus says.) When deciding whether to fill a gap or leave it unfilled, a good rule of thumb is: Is it just as difficult, for the reader to understand my fill, as it is for them to fill the gap themself? If so, then the fill isn't really helping, so omit it.

I don't know what your solution to the preceding problem looks like, but my solution is pretty elementary and maybe even tedious. It's just as difficult to read, as it was to write. In this course, a lot of our basic examples are like this. That's why I skip the proofs. It's not that the gaps can't be filled rigorously; it's that everyone might as well fill the gaps for themself.

C. Section 13 Exercise 8a. (You may use the fact that between any two real numbers there is a rational number. You may use Lemma 13.2, although we have not discussed it in class. It is similar to Lemma 13.3, which we have proved. We don't have time for every theorem. I assign this homework problem partially to get you to wrestle with Lemma 13.2 a bit.)

Epilogue: Section 16 Exercise 6 is similar to Section 13 8a. Do it, if you want more practice, but I'm not asking you to hand it in.

D. Section 13 Exercise 5. Do the basis part only; skip the sub-basis part (although it's good practice, of course).