When we write "$\mathbb{R}$" without additional decoration, we are referring either to the set $\mathbb{R}$ or the space $(\mathbb{R}, T_{\text{std}})$. Which one we mean should be clear from context. For example, if we're asking a question about continuity for a function $\mathbb{R} \to \mathbb{R}$, then we need a topology on $\mathbb{R}$, so $\mathbb{R}$ must be a space. But if we're putting a topology on $\mathbb{R}$, then $\mathbb{R}$ must be a set. Make sense? If not, ask for clarification.

A. Section 16 Exercise 1 (about subspaces of subspaces).

B. In $\mathbb{R}$, find a homeomorphism between $(0, 1)$ and an arbitrary finite open interval (a, b). Also find a homeomorphism between $(0, 1)$ and $(-\infty, \infty)$. (You may use the fact, which we have not proved yet, that continuity in $\mathbb{R}$ and its subspaces matches continuity as defined in calculus.)

C. Let $Y = \{1, 1/2, 1/3, 1/4, 1/5, \ldots\}$ and let $Z = Y \cup \{0\}$, both with their subspace topologies from $\mathbb{R}$. Is Y discrete (meaning, a space with the discrete topology)? Is Z discrete?

D. What happens if you carry out the construction of the quotient topology on Y, for a function $f : X_T \to Y$ that is not surjective?

You are being asked to hand in solutions to the four problems above. You are not asked to hand in a solution to the problem below. But maybe you want to try it anyway, as a form of studying.

Consider the surjection $f : \mathbb{R} \to [0, \infty)$ given by $f(x) = x^2$. Endow $[0, \infty)$ with the quotient topology induced by f. How does it compare to the subspace topology on $[0, \infty) \subseteq \mathbb{R}$?

Also, I invite you to consider the surjection $f : \mathbb{R} \to (-\infty, \infty)$ given by an odd-degree polynomial. What is the quotient topology, if $f(x) = x^3$? If $f(x) = x^3 + x$? If $f(x) = x^3 - x$?