

**A.** [I don't draw the diagram in these solutions, but I give you information that can easily be turned into the diagram. I also give justifications.] All of these sets are subsets or supersets of each other — rather than having interesting intersections, say. From smallest to largest:

context-free languages	[contained in P by the algorithm used in Problem B]
P	[contained in NP because every DTM is trivially an NTM]
NP	[contained in NPSPACE by space-time lemma]
PSPACE = NPSPACE	[ $\subseteq$ because every DTM is an NTM; $\supseteq$ by Savitch]
EXPTIME	[contains PSPACE by counting configurations]
EXPSPACE	[contains EXPTIME by space-time lemma]
decidable languages	[definitions of time and space complexity assume deciders]

**B.** The algorithm produces the following table. Because the start variable  $S$  appears in the upper right cell, we conclude that  $w$  is derivable from the grammar.

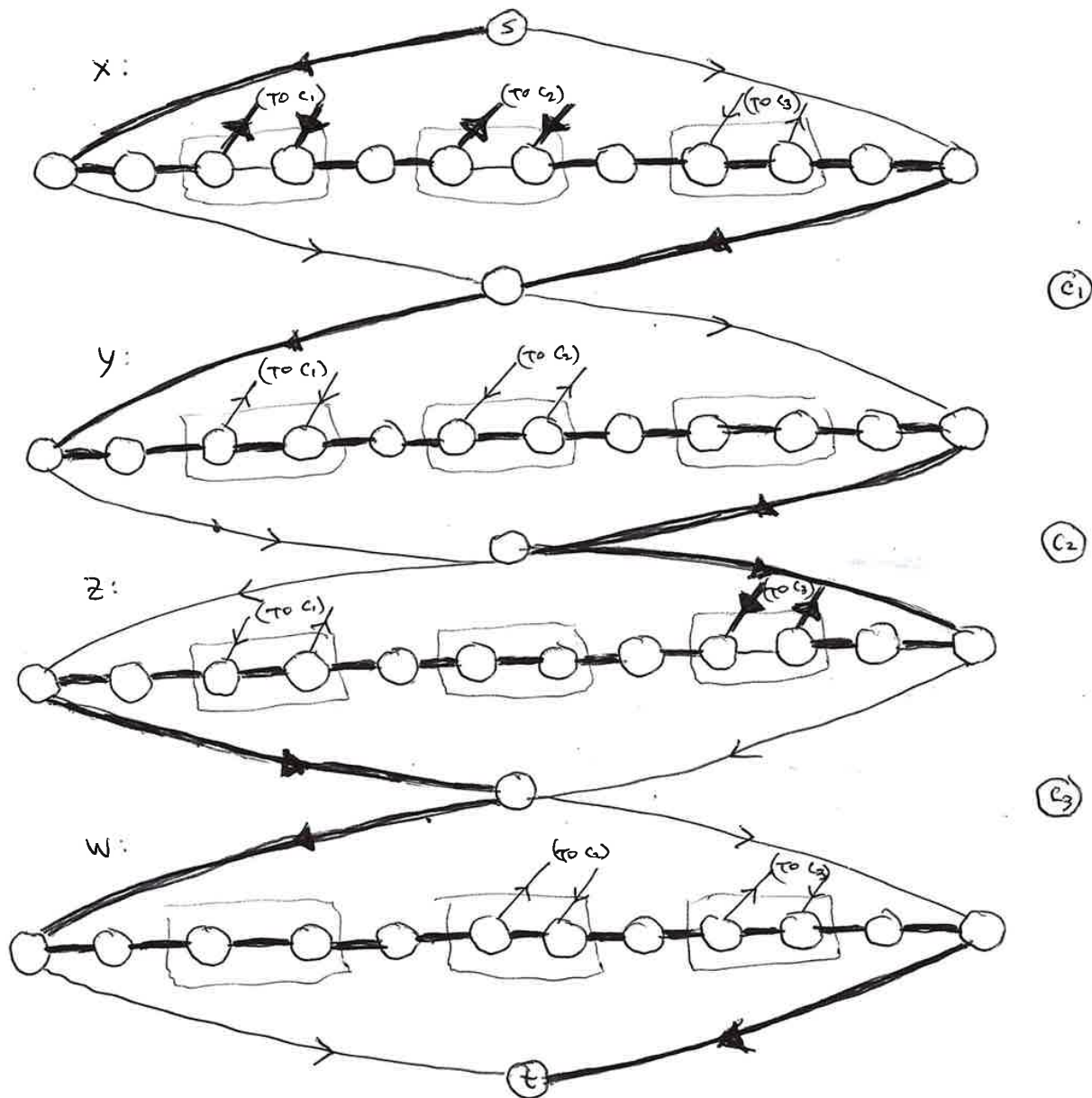
	$j = 1$	2	3	4	5
$i = 1$	$S, T, A$	$U$	[none]	[none]	$S, T$
2		$S, T, A$	$V$	$T, S$	$U$
3			$S, T, B$	$U$	[none]
4				$S, T, A$	$U$
5					$S, T, A$

**C.** The upper bound in question is  $t(n) \leq 2^{\mathcal{O}(s(n))}$ , which is the number of configurations that an  $s(n)$ -space Turing machine has. If the deterministic Turing machine is a decider, then it can't repeat a configuration, because then it would repeat indefinitely and therefore hang.

So the question is: Does nondeterminism allow a nondeterministic decider to repeat a configuration? Intuitively, you might think so, because the machine can choose to repeat once or twice and then not repeat any more. But recall that we defined a nondeterministic Turing machine to be a decider if, on every input, every branch of computation halts. So, if it is able to repeat a configuration, then some branch will indefinitely repeat that configuration, and the nondeterministic Turing machine will not be a decider.

So yes, the argument works;  $t(n) \leq 2^{\mathcal{O}(s(n))}$  for nondeterministic Turing machines as well.

**D.A.** [Here I can't avoid drawing a picture.] To keep the picture clean, I have adopted an unusual notation: The six edges marked "to  $c_1$ " attach to node  $c_1$ , and similarly for  $c_2$  and  $c_3$ . Also, edges without arrows are meant to be bi-directional.



**D.B.** Yes, the formula can be satisfied, and so there is a Hamiltonian path through  $G$  from  $s$  to  $t$ . Here's an example, also shown in bold in the diagram above.

A satisfying assignment is  $x = \text{TRUE}$ ,  $y = \text{TRUE}$ ,  $z = \text{FALSE}$ ,  $w = \text{TRUE}$ . The corresponding path traverses the  $x$  diamond left-to-right, the  $y$  diamond left-to-right, the  $z$  diamond right-to-left, and the  $w$  diamond left-to-right.

The detours to  $c_1, c_2, c_3$  are not unique. The fact that  $x$  makes the first clause TRUE allows the path to detour to  $c_1$  from the  $x$  diamond. The fact that  $x$  makes the second clause TRUE allows the path to detour to  $c_2$  from the  $x$  diamond. The fact that  $\bar{z}$  makes the third clause true allows the path to detour to  $c_3$  from the  $z$  diamond.

**E.A.** Let  $s = 0^p 110^p$ .

**E.B.** Let  $s = 0^p 10^p$  (which uses  $w = \epsilon$ ).

**E.C.** Let  $s = 0^p 1^p 20^p 1^p$ .

**E.D.** Let  $s = 0^p 10^{2p} 10^{3p}$ .

**F.** Yes,  $\neq\text{SAT}$  is  $NP$ -complete, as we now check.

First,  $\neq\text{SAT}$  is an element of  $NP$ , for approximately the same reason that  $\text{SAT}$  and  $3\text{SAT}$  are: A polynomial-time verifier could take a proposed unequal-satisfying assignment as a certificate and check whether it works.

Second,  $\neq\text{SAT}$  is  $NP$ -hard, because any  $A \in NP$  satisfies  $A \leq_p 3\text{SAT}$  (the Cook-Levin theorem, from class) and  $3\text{SAT} \leq_p \neq\text{SAT}$  (from homework).