Please write your name at the top of this page and nowhere else.

In addition to this cover page, there should be problems A–F spread over six pages.

You are allowed one sheet of paper with notes on both sides, as we have discussed. Otherwise, no notes, books, calculators, computers, etc. are allowed.

If a problem is unclear and you cannot obtain clarification, then write your interpretation of the problem, so that I can evaluate your solution relative to your interpretation. You might be penalized, if your interpretation makes the problem much easier than it should be. Certainly you should never interpret a problem in a way that renders it trivial.

Except where otherwise noted, all problems require explanation. Incorrect answers with good work often earn partial credit. Correct answers without justification rarely earn full credit. Write as if your audience is a typical classmate — not a professor. In doing so, you (hopefully) show enough detail, that I can evaluate whether you understand your arguments.

Except where otherwise directed, you may cite material (definitions, theorems, etc.) that we have defined or proved in class, in the assigned textbook readings, or in the assigned homework. You do not need to re-define or re-prove any of that material. You may not cite other material without developing it first. When in doubt, ask me.

Pictures often help both you and your reader.

You have 150 minutes (2.5 hours). Good luck. :)

In class, we have drawn a Venn diagram showing how the set of regular languages relates to the set of context-free languages, the set of decidable languages, etc.

A. Draw a Venn diagram showing how the following eight sets of languages relate to each other. I specifically want a single Venn diagram, rather than some other depiction of their relationships. You do NOT need to explain or justify your diagram, but you may explain, if you like.

1. context-free languages
2. decidable languages
3. EXPSPACE
4. EXPTIME
5. NP
6. NPSPACE
7. P
8. PSPACE

Problem B refers to this context-free grammar, which is in Chomsky normal form:

$$S \to AU \mid BV \mid a \mid b \mid \epsilon \qquad T \to AU \mid BV \mid a \mid b \qquad U \to TA \qquad V \to TB \qquad A \to a \qquad B \to b$$

B. Execute our usual dynamic programming algorithm, to determine whether the string $w = aabaa$ is derivable from the grammar above.

In class, we derived an upper bound on $t(n)$, in terms of $s(n)$, for deterministic Turing machines that are deciders.

C. Does that argument also work for non-deterministic Turing machines that are deciders?

D.A. Execute the polynomial-time mapping reduction from 3SAT to HAMPATH, on the Boolean formula $\phi = (x \vee y \vee \bar{z}) \wedge (x \vee \bar{y} \vee w) \wedge (w \vee \bar{z} \vee \bar{x})$. Your answer will probably be a large picture.

D.B. If ϕ is satisfiable, then give a satisfying assignment, and show what it corresponds to in your answer to D.A. If ϕ is not satisfiable, then explain how that unsatisfiability manifests in your answer to D.A.

Problem E is about the two pumping lemmas. In each part, give the string s, that you would use to carry out a pumping-lemma-based proof of the given claim. You do NOT need to explain or justify that string. You do NOT need carry out any more of the proof than picking the string.

E.A. $\{ww^{\mathcal{R}} : w \in \{0,1\}^*\}$ is not regular. (Here, $w^{\mathcal{R}}$ is the reverse of w.)

E.B. $\{0^m 1 w 0^m : m \geq 0, w \in \{0,1\}^*\}$ is not regular.

E.C. Over $\Sigma = \{0,1,2\}$, $\{w2w : w \in \{0,1\}^*\}$ is not context-free.

E.D. $\{0^m 1 0^{2m} 1 0^{3m} : m \geq 0\}$ is not context-free.

F. Is $\neq$SAT an NP-complete language? (Certainly your answer needs explanation.)