

A. Exercise 7.4. The given CFG is in Chomsky normal form. So you just need to execute the dynamic programming algorithm described in class and in the textbook.

Recall that

$$CLIQUE = \{\langle G, k \rangle : G \text{ is an undirected graph, } k \geq 1, \text{ and } G \text{ contains a } k\text{-clique}\}.$$

Also, for any  $k \geq 1$ , let

$$CLIQUE_k = \{\langle G \rangle : G \text{ is an undirected graph that contains a } k\text{-clique}\}.$$

In class, we will soon learn that  $CLIQUE$  is  $NP$ -complete. This fact implies that if  $CLIQUE \in P$ , then  $P = NP$ . The popular belief is that  $P \neq NP$  and hence  $CLIQUE \notin P$ .

B. Show that  $CLIQUE_k \in P$  for all  $k$ . (For the sake of the next problem, try to pin down your running time fairly precisely. By the way, the  $k = 3$  case is Exercise 7.9 in our textbook.)

C. Explain how it's possible that  $CLIQUE_k \in P$  for all  $k$ , but  $CLIQUE \notin P$ . In other words, explain why someone might think that  $(\forall k \text{ } CLIQUE_k \in P) \Rightarrow CLIQUE \in P$ , and why that argument can't be completed.