

A. Exercise 7.6, which is about closure properties of P .

B. Exercise 7.7, which is about closure properties of NP .

Is NP closed under complementation? Nobody knows, but the common suspicion is that NP is not closed under complementation. Given below are two erroneous “proofs” that NP is closed under complementation. They are extremely similar in their texts — there is only one small difference, which I have put in bold — but they are quite different in their logical errors. Explain what is wrong with each of them.

C.A. Let $A \in NP$. Then there exists an NTM N and natural number k such that $L(N) = A$ and the running time of N is $\mathcal{O}(n^k)$. Define a **TM** M that, on input w , runs N on w and outputs the opposite of what N outputs. Then $L(M) = \overline{L(N)} = \overline{A}$, and the running time of M is $\mathcal{O}(n^k)$. So $\overline{A} \in NP$.

C.B. Let $A \in NP$. Then there exists an NTM N and natural number k such that $L(N) = A$ and the running time of N is $\mathcal{O}(n^k)$. Define an **NTM** M that, on input w , runs N on w and outputs the opposite of what N outputs. Then $L(M) = \overline{L(N)} = \overline{A}$, and the running time of M is $\mathcal{O}(n^k)$. So $\overline{A} \in NP$.