

Please write your name at the top of this page and nowhere else.

In addition to this cover page, there should be problems A–D.

No notes, books, calculators, computers, etc. are allowed.

If a problem is unclear, then ask for clarification. If you cannot obtain clarification, then write your interpretation of the problem, so that I can evaluate your solution relative to your interpretation. You might be penalized, if your interpretation makes the problem much easier than it should be. Certainly you should never interpret a problem in a way that renders it trivial.

Except where otherwise noted, all problems require explanation. Incorrect answers with good work often earn partial credit. Correct answers without justification rarely earn full credit. Write as if your audience is a typical classmate — not your instructor. In doing so, you (hopefully) show enough detail, that I can evaluate whether you understand your arguments.

Except where otherwise directed, you may cite material (definitions, theorems, etc.) that we have defined or proved in class, in the assigned textbook readings, or in the assigned homework. You do not need to re-define or re-prove any of that material. You may not cite other material without developing it first. When in doubt, ask me.

Pictures often help both you and your reader.

You have 70 minutes. Good luck. :)

Let $r > 0$ be constant. Let $S = \text{image}(x)$, where $x : (-\pi, \pi) \times (-\pi, \pi) \rightarrow \mathbb{R}^4$ is given by $x(u, v) = (r \cos u, r \sin u, r \cos v, r \sin v)$. I claim that $p = (r, 0, r, 0) \in S$,

$$X = (0, 3, 0, 4) \in T_p S, \quad \text{and} \quad Y = (0, 4, 0, 3) \in T_p S.$$

A.A. Compute the 2×2 matrix g for this x .

A.B. Check that $X \in T_p S$.

A.C. Compute $X \cdot Y$ using the g -based dot product in $\mathbb{R}^2$. Show work thoroughly.

Let $\alpha : I \rightarrow \mathbb{R}^3$ be a regular curve parametrized by arc length with $\kappa(s) > 0$ for all $s \in I$. Notice that, if the binormal B is constant, then $\dot{B}$ is constant.

B. Using the Frenet-Serret equations, prove the converse: If $\dot{B}$ is constant, then B is constant.

Conceptually, *regular surface* is to *surface* as *regular curve* is to ... *curve*? But that concept of *curve*, that completes the analogy, has been missing from this course.

C.A. Draw an example of a regular curve that's not a curve. (If you don't know what I mean, then try Problem C.B first?) Explain or annotate enough, that I understand your drawing.

C.B. Give a rigorous definition of *curve* that completes the analogy above.

Each part of this problem is a true-false question. Remember to justify your answers.

D.A. If $\alpha : I \rightarrow \mathbb{R}^n$ is a regular curve, $u_0 \in I$, and the acceleration $a(u_0)$ is a scalar multiple of the velocity $v(u_0)$, then $\kappa(u_0) = 0$.

D.B. For every local parametrization $x : U \rightarrow \mathbb{R}^n$ of a surface S , the 2×2 matrix g must be invertible at every point $(u, v) \in U$.

D.C. If $f : \mathbb{R}^3 \rightarrow \mathbb{R}$ is smooth, $c \in \mathbb{R}$ is constant, and $S = \{x : f(x) = c\}$ is non-empty, then S is a surface.