

Our regular homework schedule resumes: This Day 12 homework is due on Day 14. There are three required problems, which are concrete practice of the abstract concepts that we learned today. All problems involve the surface

$$S = \{x : x_3^2 = x_1^2 + x_2^2, 0 < x_3 < 1\} \subseteq \mathbb{R}^3,$$

which is part of a cone. Here are two local parametrizations. Let $U = (0, 1) \times (-\pi, \pi)$ and $V = \{(s, t) : 0 < s^2 + t^2 < 1\}$, and let $x : U \rightarrow \mathbb{R}^3$ and $y : V \rightarrow \mathbb{R}^3$ be given by

$$x(u, v) = \begin{bmatrix} u \cos v \\ u \sin v \\ u \end{bmatrix}, \quad y(s, t) = \begin{bmatrix} s \\ t \\ (s^2 + t^2)^{1/2} \end{bmatrix}.$$

(Aside: This x is an example of what kind of parametrization, and this y is an example of what kind of parametrization?)

Problem A. What are U' , V' , and the diffeomorphism $h : U' \rightarrow V'$ such that $x = y \circ h$ on U' ? (Be careful. Hint: $F_y \circ x = F_y \circ y \circ h = \text{what?}$)

Problem B. Verify the metric transformation rule $g_x = (\partial h / \partial u)^\top g_y (\partial h / \partial u)$ by explicitly computing each quantity in terms of u and v and verifying that the two sides of the equation are equal.

Problem C. Let $R = \{x \in S : x_1 \geq 0\}$, which is half of S . What is the corresponding region Q of U , and what is the area of R as computed using x and Q ? What is the corresponding region P of V , and what is the area of R computed using y and P ?