

This Day 09 homework is due on Day 12 (Friday) because of our exam on Day 11 (Wednesday). However, the exam covers all course material up to and including this homework. So you should complete these problems before Day 11, even if you do not write up and hand in your solutions until Day 12.

There are three required problems and no optional problems. You should be able to do the calculations by hand. If you're confident that you could do them by hand, then you are welcome to use Mathematica instead, showing the brief Mathematica code required and the output that it produces.

We proved the following lemma a day or two ago in class. The following problem adds a detail that becomes important when we talk about arc length.

Lemma: Let $x : U \rightarrow \mathbb{R}^n$ be a local parametrization of a surface S . Let $\alpha : I \rightarrow \mathbb{R}^n$ be smooth such that $\alpha(I) \subseteq x(U)$. Then there exists a smooth $\beta : I \rightarrow U$ such that $\alpha(t) = x(\beta(t))$ for all $t \in I$.

Problem A. Prove that β is regular if and only if α is regular.

In class, we have defined stereographic projection F of $\mathbb{S}^2$ from the north pole onto the equatorial plane. And we have written down its quasi-inverse x , which is a local parametrization of $\mathbb{S}^2$ at every point p other than the north pole.

Problem B. Compute the 2×2 matrix g for this x . Simplify it as much as possible. Comment on how simple it is — meaning, is there a special name for a matrix that's this simple?

Problem C.A. Compute g for the spherical coordinates parametrization x from the Day 08 homework.

Problem C.B. In retrospect, which tool from today's class could have told you that answer more quickly?

Problem C.C. The point $p = (1, 0, 0)$ is in the image of this x and in the image of x from Problem B. Do the two x s give the same matrix g at that point?

Problem C.D. The point $p = (1/\sqrt{2}, 0, 1/\sqrt{2})$ is also in both images. Do the two x s give the same matrix g at that point?