

There are three required problems and no optional problems.

For the first problem, let $x : \mathbb{R}^2 \rightarrow \mathbb{R}^n$ be a linear transformation. In other words, there exists an $n \times 2$ matrix A such that

$$x(u, v) = A \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \\ \vdots & \vdots \\ A_{n1} & A_{n2} \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}.$$

Your answers to the following questions may require cases that depend on the properties of A . (We're practicing linear algebra while we study differential geometry.)

Problem A.A. Is x injective (one-to-one)? Is x surjective (onto)?

Problem A.B. Is x a regular surface, as defined in class?

For the next problem, let $x : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be given by $x(u, v) = (u^3, v)$.

Problem B.A. Is x injective? Is x surjective?

Problem B.B. Is x a regular surface?

For the next problem, let $U = (-\pi, \pi) \times \mathbb{R} \subseteq \mathbb{R}^2$. We're going to study smooth functions $x : U \rightarrow \mathbb{R}^3$, whose images are quasi-surfaces. (By "quasi-surface", I mean something that's intuitively a surface but not necessarily a *regular surface* or *surface* according to their technical definitions.) You are not allowed to cite any version of Section 2.1 Proposition 5, because we have not yet finished the proof of that theorem in class.

Problem C.A. Let $x(u, v) = (\cos u, \sin u, v)$. Sketch the quasi-surface and describe it briefly in English. And is x a regular surface?

Problem C.B. Let $x(u, v) = (v \cos u, v \sin u, v)$. (This x is intentionally similar to Problem C.A's x .) Sketch the quasi-surface, describe it briefly, and is x a regular surface?

Problem C.C. Consider the same x as in Problem C.B. Notice that $p = (1, 0, 1)$ is a point in the quasi-surface. Is x a local parametrization about p ? If not, can you restrict the domain U to a smaller open subset $V \subseteq \mathbb{R}^2$, such that the restriction of x to V is a local parametrization about p ?

Problem C.D. Repeat Problem C.C for $p = (0, 0, 0)$.