

Attempt this homework before Day 05, and hand it in at the start of class on Day 06. There are two required problems and two optional problems.

Problem A. Devise an example of a regular curve $\alpha : I \rightarrow \mathbb{R}^3$ such that $\tau = 0$ everywhere that τ is defined, but the curve does not lie in a plane.

Let $\alpha : I \rightarrow \mathbb{R}^3$ be regular, and let $u_0 \in I$ be such that $\kappa(u_0) \neq 0$. Consider the line passing through the point $\alpha(u_0)$ in the direction of $\mathbf{N}(u_0)$. Call it the “normal line” of α at $u = u_0$. It can be parametrized as $\beta(v) = \alpha(u_0) + v\mathbf{N}(u_0)$. Notice that if α is a circle then all of its normal lines pass through the center of the circle. We now prove a version of the converse.

Problem B. Let $\alpha : I \rightarrow \mathbb{R}^3$ be arc-length-parametrized with $\kappa \neq 0$ everywhere. Let $\mathbf{p} \in \mathbb{R}^3$ be a point. Suppose that all normal lines of α pass through $\mathbf{p}$. Prove that the image of α is contained in a circle centered at $\mathbf{p}$. (This problem is difficult, but it becomes much easier if you start your solution in an auspicious way. Maybe you’d like to ask me how I start my solution?)

The two problems below are optional.

Problem C. Problem 1.13 from our textbook. (We have a few ways to express κ . Which way do you think is most convenient for this problem? And what about τ ? But yes, there’s a lot of calculation here.)

Problem D. Problem 1.15, but just the τ part, because we’ve already done the κ part. (Hint: I assumed, without loss of generality, that α is parametrized by arc length. I rewrote the right side in terms of $\mathbf{T}$. At some point I used this cyclic permutation property of the cross product: $(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{w} = (\mathbf{v} \times \mathbf{w}) \cdot \mathbf{u} = (\mathbf{w} \times \mathbf{u}) \cdot \mathbf{v}.$)