Please write your name at the top of this page, and nowhere else.

In addition to this cover page, there should be 6.5 pages of problems.

You are permitted to bring a single sheet of paper, with notes on both sides, subject to our agreed-upon rules. No books, calculators, computers, etc. are allowed.

Feel free to ask clarifying questions. If a problem is unclear and you cannot obtain clarification, then write your interpretation of the problem, so that I can evaluate your solution relative to your interpretation. You might be penalized, if your interpretation makes the problem much easier than it should be. Certainly you should never interpret a problem in a way that renders it trivial.

Unless a problem says otherwise, you may cite material (definitions, theorems, etc.) that we have developed in class, in the assigned textbook readings, or in the assigned homework. You do not need to re-define or re-prove any of that material. You may not cite other material without developing it first.

If you introduce any notation, then define it before you use it. (For example, "Let A be the event that ....") Show your work, in as organized a manner as possible.

Incorrect answers with work shown often earn partial credit. Correct answers without work shown rarely earn full credit. Do simple arithmetic but not complicated arithmetic. For example, simplify $35/14$ down to $5/2$, but do not simplify $0.14921 \cdot 0.23323$ down to 0.03480025.

Write as if your audience is a typical classmate — not a professor. In doing so, you (hopefully) show enough detail, that I can evaluate whether you yourself understand your arguments.

Pictures often help both you and your reader.

You have 150 minutes (2.5 hours). Good luck. :)

Let X and Y be jointly distributed continuous random variables, so that $f_{X,Y}(x,y) = 8xy$ on the triangle with vertices $(0,0)$, $(1,0)$, and $(1,1)$, and $f_{X,Y}(x,y) = 0$ elsewhere.

A.A. Compute the marginal distribution of X. State its support and its PDF on that support.

A.B. Compute $E(Y|X = x)$.

A.C. Compute $E(Y|X)$.

A.D. Compute $V(Y|X = x)$.

B.A. Let $X \sim \text{Bern}(p)$. Compute the MGF of X from the definition. (Don't just cite it.)

B.B. Let $Y \sim \text{Binom}(n, p)$. Use the MGF of X to find the MGF of Y. (Don't just cite it.)

B.C. Use the MGF of Y to compute the expectation of Y. (Don't just cite it.)

Let $X_1, X_2, X_3, \ldots$ be the inter-arrival times in a Poisson process with rate λ. Let $S_n = X_1 + X_2 + \cdots + X_n$ be the nth arrival time.

C.A. What, if anything, does the central limit theorem say about S_n? If you discuss a probability distribution, then be sure to specify its parameters, in terms of the information given above.

C.B. In the quoted sentence below, fill in the three blanks, so that the sentence is correct and the third blank is not close to 0 or 1. Use the extra space to explain, if you like. "The probability that S_n is between

_____

and

_____

is approximately

_____."

In one sector of the economy, there are n companies. Let $X_1, X_2, \ldots, X_n$ be their profits next year. Let $S_n = X_1 + \ldots + X_n$ be the total profit of that sector next year. For simplicity, assume that the profits are identically distributed, each with expectation μ and variance σ^2.

D.A. Do policy makers want $E(S_n)$ and $V(S_n)$ to be great or small? Why?

D.B. Assuming that the profits are independent, what are $E(S_n)$ and $V(S_n)$?

D.C. No longer assuming that profits are independent, how bad could $V(S_n)$ be? (Hint: You may assume that $\text{Cov}(X, Y) \leq \text{SD}(X) \cdot \text{SD}(Y)$, for any random variables X and Y.)

D.D. Should policy makers encourage independence or dependence? Discuss.

E. A certain disease affects 1% of people. A screening test has a false negative rate of 8%, meaning that 8% of people with the disease test negative for it. The test has a false positive rate of 2%, meaning that 2% of people without the disease test positive for it. You test negative for this disease. (Congratulations!) What then is the probability that you have the disease?