Remark on Problem A: In all parts of this problem, a question of the form, "What is the probability that..." is definitely wrong. For starters, the possible answers are the interval $[0, 1]$, and none of the distributions below have support $[0, 1]$. (What if we were discussing a distribution that did have that support, such as $\text{Unif}(0, 1)$? Then would the question be correct? Possibly, but very probably not. Why?)

A.A. Considering only the next car to go by, how many of those cars are blue? The answer is probably 0 but maybe 1. The answer is $X \sim \text{Bern}(0.22)$.

A.B. How many non-blue cars will go by, before we see our first blue car? The answer is $X \sim \text{Geom}(0.22)$.

A.C. How many non-blue cars will go by, before we see 10 blue cars? The answer is $X \sim \text{NBin}(10, 0.22)$.

A.D. Of the next 10 cars that go by, how many will be blue? The answer is $X \sim \text{Binom}(10, 0.22)$.

A.E. My answers to parts B, C, and D assume, that there are so many cars in the world, that it doesn't matter, that there are finitely many cars and hence successive cars are not exactly independent. If we want to be exact, let c be the number of cars, so that $0.22c$ is the number of blue cars and $0.78c$ is the number of non-blue cars. (I assume that both are integers.) Now I ask: Of the next 10 cars that go by, how many will be blue? The answer is $X \sim \text{HGeom}(0.22c, 0.78c, 10)$.

B.A. Then $X + Y \sim \text{Binom}(2, p)$, by the story of the binomial distribution, which we've covered in class.

B.B. Then $X + Y \sim \text{NBin}(2, p)$, by the story of the negative binomial distribution, which we've covered in class.

B.C. Then $X + Y \sim \text{NBin}(2r, p)$, by the story of the negative binomial distribution again.

B.D. Then $X + Y \sim \text{Binom}(2n, p)$, by the story of the binomial distribution again. Also, this is a special case of a homework problem.

B.E. It is tempting to say that $X + Y \sim \text{HGeom}(2m, 2b, 2n)$, because we are drawing $2n$ balls from a total of $2m$ maize balls and $2b$ blue balls. However, this answer is incorrect, because it introduces dependence that contradicts the independence of X and Y. As far as I can tell, this $X + Y$ does not follow any of our usual distributions.

C.A. Let A, B, C be the events that the capacitor was made by AceTech, Bronfels, Capacicorp, respectively. Let D be the event that it is defective. Then, by the law of total probability,

$$\begin{aligned} P(D) &= P(D|A)P(A) + P(D|B)P(B) + P(D|C)P(C) \\ &= 0.0008 \cdot 0.41 + 0.0003 \cdot 0.32 + 0.0012 \cdot 0.27. \end{aligned}$$

C.B. Now let A, B, C be the events that all of my phone's capacitors were made by those companies, respectively. We assume that their probabilities are as above. Let N be the event that none are defective. Then, assuming defects are independent (because we have no information about any particular dependency),

$$\begin{aligned} P(N) &= P(N|A)P(A) + P(N|B)P(B) + P(N|C)P(C) \\ &= (1 - 0.0008)^{14} \cdot 0.41 + (1 - 0.0003)^{14} \cdot 0.32 + (1 - 0.0012)^{14} \cdot 0.27. \end{aligned}$$

C.C. Let the events A, B, C, D be as in part A of the problem. By Bayes's theorem and part A,

$$\begin{aligned} P(C|D) &= \frac{P(D|C)P(C)}{P(D)} \\ &= \frac{0.0012 \cdot 0.27}{0.0008 \cdot 0.41 + 0.0003 \cdot 0.32 + 0.0012 \cdot 0.27}. \end{aligned}$$

D.A. Let \$$X$ be the casino's earnings from a single play of the game. Then the support of X is $\{-1, 1\}$, and $P(X = -1) = 9/19$ and $P(X = 1) = 10/19$. The expectation of X is

$$E(X) = -1 \cdot \frac{9}{19} + 1 \cdot \frac{10}{19} = \frac{1}{19}.$$

The casino's daily earnings are $X_1 + \cdots + X_n$, where each X_i is distributed just as the X above. By linearity of expectation, the average daily earnings are

$$E(X_1 + \cdots + X_n) = E(X_1) + \cdots + E(X_n) = nE(X) = n/19.$$

Here's a slightly more clever strategy. Let $Y \sim \text{Bern}(10/19)$, so that $X = 2Y - 1$. Then we can compute $E(X)$ (and $V(X)$, below) by manipulating $E(Y)$ (and $V(Y)$), which we have memorized. I leave the details to you.

D.B. Continuing the notation from part A of this problem, we have

$$E(X^2) = (-1)^2 \cdot \left(\frac{9}{19}\right) + 1^2 \cdot \left(\frac{10}{19}\right) = 1.$$

Therefore

$$V(X) = E(X^2) - E(X)^2 = 1 - \left(\frac{1}{19}\right)^2 = \frac{360}{361}.$$

Assume that plays of roulette are independent. (If you know roulette, then you know this to be true. If you don't know roulette, but are instead worried about dependence, then you have no information about that dependence and no way to complete the problem.) Then

$$V(X_1 + \cdots + X_n) = V(X_1) + \cdots + V(X_n) = nV(X) = n\frac{360}{361}.$$

Epilogue: Thus the standard deviation of the daily earnings is slightly less than $\sqrt{n}$. If $n = 10{,}000$, for example, then the average is about 526 and the standard deviation is about 100. Once we learn the central limit theorem, we will better appreciate how this makes running roulette games a viable business.

E.A. Usually, when two dice are rolled, there are 36 outcomes, each with probability 1/36. In this case, however, there are only 35 outcomes for a valid roll, each with probability 1/35. The support of X is $\{1, 2, 3, 4, 5, 6\}$. The probability that $X = 1$ is

$$P(X = 1) = \sum_{k=1}^{6} P(X = 1, Y = k) = 0 + \frac{1}{35} + \frac{1}{35} + \frac{1}{35} + \frac{1}{35} + \frac{1}{35} = \frac{5}{35}.$$

Meanwhile,

$$P(X = 2) = \sum_{k=1}^{6} P(X = 2, Y = k) = \frac{1}{35} + \frac{1}{35} + \frac{1}{35} + \frac{1}{35} + \frac{1}{35} + \frac{1}{35} = \frac{6}{35}.$$

And $P(X = k) = P(X = 2)$ for $k = 3, 4, 5, 6$.

E.B. No, X and Y are not independent, because knowing that $X = 1$ tells us that $Y \neq 1$. More rigorously, Y has the same distribution as X, and

$$P(X = 1, Y = 1) = 0 \neq \frac{5}{35} \cdot \frac{5}{35} = P(X = 1) \cdot P(Y = 1).$$