In addition to this cover page, there should be five pages of problems.

No notes, books, calculators, computers, etc. are allowed. You might find the following facts from calculus helpful. First, for all real numbers r such that $|r| < 1$,

$$1 + r + r^2 + r^3 + \cdots = \sum_{k=0}^{\infty} r^k = \frac{1}{1-r}.$$

Second, for all real numbers x,

$$e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!} = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \cdots .$$

Feel free to ask clarifying questions. If a problem is unclear and you cannot obtain clarification, then write your interpretation of the problem, so that I can evaluate your solution relative to your interpretation. You might be penalized, if your interpretation makes the problem much easier than it should be. Certainly you should never interpret a problem in a way that renders it trivial.

You may cite material (definitions, theorems, etc.) that we have developed in class, in the assigned textbook readings, or in the assigned homework. You do not need to re-define or re-prove any of that material. You may not cite other material without developing it first.

If you introduce any notation, then define it before you use it. (For example, "Let A be the event that ....") Show your work, in as organized a manner as possible.

Incorrect answers with work shown often earn partial credit. Correct answers without work shown rarely earn full credit. Do simple arithmetic but not complicated arithmetic. For example, simplify $35/14$ down to $5/2$, but do not simplify $0.14921 \cdot 0.23323$ down to 0.03480025.

Write as if your audience is a typical classmate — not a professor. In doing so, you (hopefully) show enough detail, that I can evaluate whether you yourself understand your arguments.

You have 60 minutes. Good luck. :)

My daughter and I are sitting at a roadside restaurant, watching cars go by. It seems that 22% of cars are blue, 31% are red, 15% are white, and the remainer are various other colors. Within this context, for each of the following distributions, ask a question in plain English, such that the answer is a random variable that follows the given distribution. State the parameter values.

A.A. Bern

A.B. Geom

A.C. NBin

A.D. Binom

A.E. HGeom

In each part of this problem, you are given independent X and Y and asked: What is the distribution of $X + Y$? If the answer is one of our standard distributions, then say which one, including its parameter values; if not, then explain why not. If we know the answer from class or homework, then say so; if not (or if in doubt), then explain your answer.

B.A. $X, Y \sim \text{Bern}(p)$

B.B. $X, Y \sim \text{Geom}(p)$

B.C. $X, Y \sim \text{NBin}(r, p)$

B.D. $X, Y \sim \text{Binom}(n, p)$

B.E. $X, Y \sim \text{HGeom}(m, b, n)$

Phones contain tiny devices called capacitors, which are manufactured by three companies. AceTech supplies 41% of phone capacitors, and 0.08% of its capacitors are defective. For Bronfels, these numbers are 32% and 0.03%. For Capacicorp, they are 27% and 0.12%.

C.A. What is the probability that a randomly selected capacitor is defective?

C.B. Your phone uses 14 capacitors, all made by a single company (but you don't know which). What is the probability that none of your capacitors are defective?

C.C. Your phone's speaker dies. A technician tells you that it died because a certain capacitor was defective. What is the probability that that capacitor was made by Capacicorp?

The game roulette is played at a casino. Each time it is played, the casino has a 20/38 probability of gaining \$1 from the player, and a 18/38 probability of losing \$1 to the player. Suppose that roulette is played n times each day.

D.A. On average, how much money does the casino make from roulette each day?

D.B. What is the variance in the casino's daily earnings from roulette?

In a certain game, two dice are rolled. If they are not both 1, then the roll is *valid*. If they are both 1, then the roll is *invalid*; the dice are rolled again, until a valid roll occurs. Let X and Y be the results of the two dice in a valid roll.

E.A. What is the PMF of X?

E.B. Are X and Y independent?