A.A. The sample space is $\{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$.

A.B. There are eleven outcomes; one of them is 4.

A.C. An example of an event is $\{3, 5, 7, 9, 11\}$. In other words, the event is, "The result is odd."

A.D. "What is the probability that the result is odd?"

B. Because the probabilities must sum to 1, we have

$$\begin{aligned} 1 &= P(0) + \frac{2}{3}P(0) + \left(\frac{2}{3}\right)^2 P(0) + \left(\frac{2}{3}\right)^3 P(0) + \cdots \\ &= \sum_{k=0}^{\infty} \left(\frac{2}{3}\right)^k P(0) \\ &= P(0) \sum_{k=0}^{\infty} \left(\frac{2}{3}\right)^k \\ &= P(0) \cdot \frac{1}{1 - 2/3} \\ &= \frac{P(0)}{1/3} \\ &= 3P(0). \end{aligned}$$

Therefore $P(0) = 1/3$.

C. There are 15 letters overall, but 15! is an overcount of the answer, because the two "c"s are indistinguishable, as are the three "l"s, the three "e"s, and the two "o"s. Correcting the overcount, we have

$$\frac{15!}{2!3!3!2!}.$$

D. This is a Bose-Einstein-style problem. Imagine an array of $15 + 2 = 17$ cells. I pick two of the cells to hold dividers. The dividers divide the remaining 15 cells into three chunks. I color the cells in the first chunk red, the cells in the second chunk green, and the cells in the third chunk blue. In this way, I have assigned a color to each of the 15 balls. The number of ways to allocate the dividers is the answer to the problem:

$$\binom{15+2}{2}.$$

E. [This problem is taken nearly verbatim from birthday.R, which we discussed in class.] This is a birthday-style problem. Let D be the event that the pieces of information land in distinct boxes. Then

$$P(D) = \frac{m \cdot (m-1) \cdot (m-2) \cdot \cdots \cdot (m-(k+1))}{m^k} = \prod_{i=0}^{k-1} \frac{m-i}{m} = \prod_{i=0}^{k-1} \left(1 - \frac{i}{m}\right).$$

When $k \ll m$, this probability can be approximated as

$$P(D) \approx e^{(1-k)k/(2m)}.$$

F. [This is a medical screening problem in disguise.] Let B be the event that Gnorp's code is buggy, and let T be the event that it tests positive — that is, Cheez says that it's buggy. We are told that $P(B) = 14/100$, so $P(B^c) = 86/100$. We are also told that $P(T|B^c) = 3/100$ and $P(T^c|B) = 8/100$. We start filling in a contingency table. Here are two steps:

	T	T^c	
B		$\frac{8}{100} \cdot \frac{14}{100}$	$\frac{14}{100}$
B^c	$\frac{3}{100} \cdot \frac{86}{100}$		$\frac{86}{100}$
			1

	T	T^c	
B		$\frac{8}{100} \cdot \frac{14}{100}$	$\frac{14}{100}$
B^c	$\frac{3}{100} \cdot \frac{86}{100}$	$\frac{86}{100} \cdot \frac{97}{100}$	$\frac{86}{100}$
		$\frac{8}{100} \cdot \frac{14}{100} + \frac{86}{100} \cdot \frac{97}{100}$	1

That's enough to compute what we want, which is

$$P(B^c|T^c) = \frac{P(B^cT^c)}{P(T^c)} = \frac{\frac{86}{100} \cdot \frac{97}{100}}{\frac{8}{100} \cdot \frac{14}{100} + \frac{86}{100} \cdot \frac{97}{100}} = \frac{86 \cdot 97}{8 \cdot 14 + 86 \cdot 97}.$$

[The answer is nearly 99%. Congratulations? But keep in mind that I made up the numbers. They might not reflect real-world AI performance.]