

In addition to this cover page, there should be five pages of problems.

No notes, books, calculators, computers, etc. are allowed. You might find the following facts from calculus helpful. First, for all real numbers r such that $|r| < 1$,

$$1 + r + r^2 + r^3 + \dots = \sum_{k=0}^{\infty} r^k = \frac{1}{1-r}.$$

Second, for all real numbers x ,

$$e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!} = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \dots.$$

Feel free to ask clarifying questions. If a problem is unclear and you cannot obtain clarification, then write your interpretation of the problem, so that I can evaluate your solution relative to your interpretation. You might be penalized, if your interpretation makes the problem much easier than it should be. Certainly you should never interpret a problem in a way that renders it trivial.

You may cite material (definitions, theorems, etc.) that we have developed in class, in the assigned textbook readings, or in the assigned homework. You do not need to re-define or re-prove any of that material. You may not cite other material without developing it first.

If you introduce any notation, then define it before you use it. (For example, “Let A be the event that”) Show your work, in as organized a manner as possible.

Incorrect answers with work shown often earn partial credit. Correct answers without work shown rarely earn full credit. Do simple arithmetic but not complicated arithmetic. For example, simplify $35/14$ down to $5/2$, but do not simplify $0.14921 \cdot 0.23323$ down to 0.03480025 .

Write as if your audience is a typical classmate — not a professor. In doing so, you (hopefully) show enough detail, that I can evaluate whether you yourself understand your arguments.

You have 60 minutes. Good luck. :)

Problem A tests your knowledge of basic vocabulary, so answer each part precisely. In all four parts, the experiment is: Roll two dice and add their results to get a single number.

A.A. What is the sample space?

A.B. Give an example of an outcome.

A.C. Give an example of an event.

A.D. Ask a question, whose answer is a real number between 0 and 1. (You do not need to compute that number.)

B. In a certain experiment, the set of possible results is $\{0, 1, 2, 3, \dots\}$. A 1 is $2/3$ as probable as a 0. A 2 is $2/3$ as probable as a 1. A 3 is $2/3$ as probable as a 2. And so on.

What is the probability of the result being 0?

C. How many words (valid or invalid) can be produced by re-arranging the letters of the word “carletoncollege”? Keep in mind, for example, that switching the two “o”s does not produce a new word.

D. You have a red box, a green box, and blue box. You drop 15 white balls into the boxes randomly. So, for example, your result might be: 4 balls in the red box, 2 in the green box, and 9 in the blue box. How many different results are there?

E. In computer science, a hash table is a common *data structure* — a way of organizing information inside the computer’s memory. Essentially, the hash table consists of m boxes, into which we want to place $k < m$ pieces of information. The hash table works fastest when there are no collisions — that is, when no two pieces of information land in the same box. So what is the probability of no collisions?

F. In your job as a software developer, your manager asks you to write code using an AI known as Gnorp. You object that 14% of code units produced by Gnorp are known to be buggy. Your manager has a simple solution: Ask another AI, called Cheez, whether the code produced by Gnorp is buggy. You object that Cheez incorrectly reports 3% of bug-free code to be buggy, and incorrectly reports 8% of buggy code to be bug-free. Your manager replies, “Do it anyway. Everybody’s using AI. We can’t afford to be left behind.”

So you use Gnorp to write a code unit, and then Cheez tells you that it’s bug-free. What’s the probability that the code unit is bug-free?