There are three problems from the book, and three additional problems labeled A, B, C.

Exercise 4.29

Exercise 4.33

Exercise 4.35

A. Upon showing up for a party, n people drop their coats on a bed. By the end of the party, the coats are all mixed up. Upon leaving, each person takes one of the coats uniformly randomly. Some people are lucky, in that they end up with their own coat. Let X be the number of people who are lucky in this way. Find $E(X)$.

B. For the example from the end of class (X a die roll, $Y = X - 7/2$, and $Z = Y^2$), compute $\mathrm{Cov}(Y, Z)$ manually, by first computing $E(Y)$, $E(Z)$, and $E(YZ)$. (The goal of this problem is for you to have practice doing basic computations, and for you to see how the symmetry, that I claimed in class, plays out in the arithmetic.)

The goal of Problem C is to prove that $-1 \leq \mathrm{Corr}(X, Y) \leq 1$, as was claimed in class. Here, "prove" means "give a convincing explanation of the fact". Your explanation will be mostly algebra, using properties of variance, standard deviation, and covariance. To help you along, I give you several "stepping stones". Let $X' = X/\mathrm{SD}(X)$ and $Y' = Y/\mathrm{SD}(Y)$.

C.A. Show that $\mathrm{Cov}(X', Y') = \mathrm{Corr}(X, Y)$.

C.B. What are $V(X')$ and $V(Y')$?

C.C. Compute $V(X' + Y')$ and $V(X' - Y')$.

C.D. Conclude that $-1 \leq \mathrm{Corr}(X, Y) \leq 1$, as claimed. (Hint: Variances must be non-negative.)