Remember that you are expected to attempt this homework before Day 04, but that you hand it in on Day 05. There are three problems from the book and another two problems labeled A and B. For the book problems, it helps to know two facts from calculus: First, if r is a number such that $|r| < 1$, then

$$1 + r + r^2 + r^3 + \cdots = \sum_{k=0}^{\infty} r^k = \frac{1}{1-r}.$$

Infinitely long sums, such as the one above, are called *series*. Second, the function e^x is defined as the series

$$e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!} = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \cdots .$$

Exercise 1.40 (about the max of two dice)

Exercise 1.43 (about a sample space with infinitely many outcomes)

Exercise 1.44 (about another infinite sample space)

For Problem A, recall that there are $\binom{d+2}{2}$ monomials of degree d in the variables x, y, z.

A.A. Show algebraically that

$$\sum_{i=0}^{d} \sum_{j=0}^{d-i} 1 = \binom{d+2}{2}.$$

In doing so, you might find this algebraic identity helpful:

$$\sum_{k=0}^{n} k = \frac{n(n+1)}{2}.$$

A.B. Explain *why* the double summation from part A answers the monomial counting problem. (I'm looking for something quite different from "Because part A shows that they are numerically equal." I'm looking for a "story" that conveys intuition. In other words, I'm trying to guide you through another way to conceptualize the counting problem.)

Problem B ponders the question: How big must the jackpot be, for playing PowerBall to make rational sense? Let's agree that it makes sense as soon as the average payout exceeds the set ticket cost (which is \$2). In other words, we are ignoring any enjoyment that might arise from playing.

B.A. Using simulation.R, try various values of jackpots, until you find the answer to the nearest million dollars. Use the exact calculation (`sum(payouts * probs)`) rather than simulating a gazillion plays.

B.B. In real life, why does playing PowerBall at that jackpot level *still* not make rational sense? (There is a simple, strong reason. I can imagine a couple of other reasons, which are subtler and arguably less important.)

Epilogue: Correcting for the effect alluded to in Problem B.B makes the problem of “pricing” PowerBall drastically more difficult. Let’s not do it.

By the way, I have fixed the `mod` problem that arose in our discussion of simulation.R. If you download the new version, it should work correctly. Let me know, if not. In any event, the `mod` thing is not needed to solve Problem B.