You have 70 minutes.

No notes, books, calculators, computers, etc. are allowed.

Show all of your work, in as organized a manner as possible. Incorrect answers with solid work often earn partial credit. Correct answers without explanatory work rarely earn full credit.

Define any notation that you introduce. For example, if you write "$P(A)$", but neither I nor you have defined an event A, then that writing is not good.

Perform as much algebraic simplification as you can. Do not bother to do non-trivial arithmetic unless it is specifically requested. Mark your final answer clearly.

Good luck.

A. Here's a simple experiment: Roll a fair, six-sided die. Let X be the result. The following questions test your knowledge of basic vocabulary, so answer them precisely.

A.A. What is the sample space?

A.B. Give me an example of an outcome.

A.C. Give me an example of an event.

A.D. What is the name of the distribution of X?

A.E. What is the PMF of X?

A.F. What is the PMF of $X + X$?

B. Ever since we were kids, my brother has been interested in rigging dice so that they roll 6 more than 1/6 of the time. (There are various ways to do it: sand off edges, insert a lead weight, etc.) While visiting him one day, I see ten dice on his coffee table. I pick one up and immediately roll three 6s. So I ask him, "Is this die rigged?" Without looking up from the beer that he's brewing, he responds, "One of the dice on that table is rigged so that 28% of the time it rolls 6. The other nine dice are fair." What is the probability that I rolled the rigged die?

C. On these TRUE-FALSE questions there are four valid answers. If the correct answer is TRUE, then TRUE earns 3 points, TRUISH earns 2 points, FALSISH earns 1 point, and FALSE earns 0 points. If the correct answer is FALSE, then these point values are of course reversed. Do not write just T or F; write your answer completely and clearly. No explanation is needed.

C.A. If A and B are any two events such that $P(A) > 0$, $P(B) > 0$, and $A \cap B = \emptyset$, then A and B are independent.

C.B. If A and B are any two events such that $P(A) > 0$, $P(B) > 0$, and $P(A|B) = P(A)$, then A and B are independent.

C.C. It is possible to have events A, B, and C such that $P(A|B) > P(B|C) > P(C|A)$.

C.D. There are 52^5 hands of five cards.

C.E. There are $\binom{13}{1}\binom{4}{3}\binom{12}{1}\binom{4}{2}$ five-card hands that are full houses. (A *full house* occurs when three cards are of equal rank (9 or king, for example) and the other two cards are of equal rank.)

D. On the Olympic Peninsula in Washington, F frogs are infected with a chytrid fungus and N frogs are not infected. For your research project, you roam the peninsula, examining frogs and recording whether they are infected. Within this context, ask a question whose answer is a random variable distributed according to each of the following distributions. Also state how the distribution's parameters depend on F, N, and anything else that's relevant.

D.A. Geometric:

D.B. Hypergeometric:

D.C. Binomial:

D.D. Negative binomial:

E. There are 18 faculty in the Math-Stats Department. Each morning I select one of them (uniformly randomly) and take a cup of coffee to them in their office. On the 55th day of this routine, I am running late, so I take the coffee directly to our department meeting and ask, "Who hasn't had 3 cups of coffee from me yet?" And it turns out that every faculty member has had exactly 3 cups. Wow! What are the chances of that?