

You have 70 minutes.

No notes, books, calculators, computers, etc. are allowed.

Show all of your work, in as organized a manner as possible. Incorrect answers with solid work often earn partial credit. Correct answers without explanatory work rarely earn full credit.

In an integration problem, first be utterly clear about which integral you are computing. Then try to compute the integral. If you do not have time to finish, then be utterly clear about what strategy you would use, if you had more time.

Perform as much algebraic simplification as you can. Do simple arithmetic, but don't bother to do complicated arithmetic. Mark your final answer clearly.

Remember that I am grading your paper, not you. Make sure your paper says exactly what you want it to say.

Good luck. :)

A. Consider $f(x, y) = x^2y$ and the triangle with vertices $(1, 1)$, $(2, 1)$, and $(2, 2)$.

A.A. Write, but do not compute, a $dx dy$ integral for f over the triangle.

A.B. Write, but do not compute, a $dy dx$ integral for f over the triangle.

A.C. Compute whichever of the two integrals you prefer.

B. A geophysicist models the Earth as a ball of radius $R = 6.37 \cdot 10^6$ m, whose density is Ae^{-Bs} kg/m³ at distance s m from the center, with $A = 12.9 \cdot 10^3$ and $B = 5.91 \cdot 10^{-4}$. Help the geophysicist compute the mass of the Earth.

C. A city occupies a circular region of radius R km. The air over the city is polluted, with more pollution close to the ground than higher up in the sky. An environmental engineer models the pollution density as $Az^{-1/2}$ g/km³, where A is a constant. Help the engineer calculate the total mass of pollution over the city, up to an altitude of 250 m.

D. Consider the tetrahedron bounded by the planes $x = 0$, $z = 0$, $y + z = 2$, and $-3x + 3y + z = 0$. (A *tetrahedron* is a pyramid-like region whose boundary consists of four triangles.) By the way, its four vertices are $(0, 0, 0)$, $(2, 2, 0)$, $(0, 2, 0)$, and $(0, -1, 3)$. Integrate z over it.