You have 150 minutes.

No notes, books, calculators, computers, etc. are allowed.

You are expected to show all work, in as organized a manner as possible. (Incorrect answers with solid work often earn partial credit. Correct answers without explanatory work rarely earn full credit.) Perform as much algebraic simplification as you can. Do not bother to do non-trivial arithmetic. Mark your final answer clearly.

You may cite without proof any result discussed in class, the assigned textbook sections, or the assigned homework, unless doing so trivializes the problem. If you do not know what you may cite, then ask me for clarification.

Good luck. :)

Oh, and here are two definitions and two theorems, which are relevant to Problem G.

$$\begin{aligned} \sinh x &= \frac{1}{2}(e^x - e^{-x}), \\ \cosh x &= \frac{1}{2}(e^x + e^{-x}), \\ \frac{d}{dx}\sinh x &= \cosh x, \\ \frac{d}{dx}\cosh x &= \sinh x. \end{aligned}$$

A. Find a power series solution $y = y(x)$ to the differential equation $y' - x^2 y = 0$. If the answer is a function that you recognize, then explain further.

B. Let E be solid hemisphere described by $x^2 + y^2 + z^2 \leq 1$ and $z \geq 0$. The integral $\iiint_E z \, dV$ is related to the center of mass of E. Compute this integral.

C. Does $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n\sqrt{n}-1}$ diverge or converge?

D. A charged particle is fixed at the origin in three-dimensional space. We wish to move another charged particle toward it. When the second particle is at position $\vec{r} = (x, y, z)$, the force on it is $\vec{F} = \nabla f = \frac{c}{|\vec{r}|^3}\vec{r}$, where $f = -\frac{c}{|\vec{r}|}$ and c is a positive constant.

D.A. How much work is needed to move the second particle from very far away ($|\vec{r}| \approx \infty$) to a distance of 1 from the origin ($|\vec{r}| = 1$)?

D.B. How much work is needed to move the second particle from there to the origin?

E.A. Let $a_n = \sqrt[n]{n}$ for $n \geq 1$. Does the sequence $\{a_n\}_{n=1}^{\infty}$ converge? If so, then to what?

E.B. Does the series $\displaystyle\sum_{n=1}^{\infty} a_n$ converge?

F. Consider the surface $z = x^2 + y^2$ and the plane $z = 8$. They intersect along a curve. Give a parametrization of that curve. Include the interval, from which the parameter takes its values.

G. Find the Taylor series for $f(x) = \sinh x$ at $a = 0$. Also find its interval of convergence.