You have 60 minutes.

No notes, books, calculators, computers, etc. are allowed.

Show all of your work, in as organized a manner as possible. Incorrect answers with solid work often earn partial credit. Correct answers without explanatory work rarely earn full credit.

Perform as much algebraic simplification as you can. Do not bother to do non-trivial arithmetic. Simplify expressions such as $\sin 30°$ but not expressions such as $\sin 35°$. Mark your final answer clearly.

Throughout this exam, saying that a function f is *smooth* means that it is infinitely differentiable: f is differentiable, its first derivatives are differentiable, its second derivatives are differentiable, etc.

Good luck.

A. Let $\vec{v} = \langle 2, 1, -1, 1 \rangle$ and $\vec{w} = \langle 1, 3, 3, 2 \rangle$. Calculate these four quantities.

$\vec{v} + \vec{w} =$

$4\vec{v} =$

$\vec{v} \cdot \vec{w} =$

$\mathrm{proj}_{\vec{w}} \vec{v} =$

B. The *gravitational potential* at a point $\vec{x} = (x, y, z)$, relative to a mass M at the origin, is defined to be $f(\vec{x}) = -GM/|\vec{x}|$, where G is a constant. The acceleration experienced on a body at location $\vec{x}$ is then $-\nabla f$ evaluated at $\vec{x}$. Show that this acceleration vector points toward the origin and has length $GM/|\vec{x}|^2$.

C1. Let $f(t)$ be a smooth function and $\vec{r}(t) = (t, f(t), 0)$. Show that $\kappa(t) = \dfrac{|f''(t)|}{(1 + (f'(t))^2)^{3/2}}$.

C2. What is the torsion $\tau(t)$ of this curve $\vec{r}$, and why? (Hint: No calculation is needed.)

D. Assume that the Earth is a spherical ball of radius 6371 km. Johannesburg, South Africa is at 26°S latitude, 28°E longitude. Phnom Penh, Cambodia is at 12°N latitude, 105°E longitude. Compute the distance between them along the surface of the Earth.

E. Setting $\rho = 1$, the spherical coordinate expressions $x = \sin\phi\cos\theta$, $y = \sin\phi\sin\theta$, $z = \cos\phi$ parametrize the unit sphere $x^2+y^2+z^2 = 1$ in three dimensions, using two angles ϕ and θ. How can we similarly parametrize the unit sphere $x^2 + y^2 + z^2 + w^2 = 1$ in four dimensions, using three angles ϕ, θ, and ψ?

F1. Let $f(x,y) = \dfrac{x^3 y}{x^6 + y^2}$. Compute the limit of f at $(0,0)$ along *every* line through $(0,0)$.

F2. Compute the limit of f at $(0,0)$ along the curve $y = x^3$. So what is $\displaystyle\lim_{(x,y)\to(0,0)} f$?

G. Is there any function $f(x, y)$ such that $\nabla f = \langle x^2 \cos(x^3 y^3), y^2 \sin(x^3 y^3)\rangle$? If so, find f. If not, explain why.

H. Find a polar equation for the circle of radius R centered at $(R, 0)$. Simplify.