An equation is a statement

Here is a snippet of writing taken from a student's math homework.

$18/2 = 9.$

When we write such an equation, we are saying that the number 18/2 and the number 9 are actually the same number, written in two different ways. Similarly, when we write

$\tan^2 x + 1 = \sec^2 x.$

we are saying that the function $\tan^2 x + 1$ and the function $\sec^2 x$ are the same function — in other words, for every number x, the number $\tan^2 x + 1$ is the same as the number $\sec^2 x$ (when they are defined at all).

An equation is a statement — a sentence, complete with a subject and a verb, ending in a period, that makes a definite claim that two objects are equal. You can read the sentence aloud in English and make sense of it. For example, consider the snippet

$2(3.75 + 0.25) = 8 \cos 0.$

You can rewrite this equation in plain text as

2, times the quantity 3.75 plus 0.25, equals 8 times the cosine of 0.

This statement makes sense, as a grammatical English sentence. The subject is "2 times the quantity 3.75 plus 0.25". The verb is "equals". The object is "8 times the cosine of 0". The sentence ends in a period, like any other declarative English sentence.

(The subject and the object in the sentence above are long, because they contain subclauses. Before a lot of our mathematical notation was standardized in the 1700s, math really was written out in plain text like this. Reading it now is painful. Symbolic notation, although it can be intimidating to the uninitiated, is superior to plain text in its conciseness and precision.)

So far we've translated from symbols into plain text. Now let's try going the other way:

The sine of the quantity x plus y equals the sine of x times the cosine of y, plus the cosine of x times the sine of y.

This is a standard English sentence, just like those you write in your English or history papers. The variables x and y might look odd in a sentence, but keep in mind that they are really just names like "Anna" or "Babatope" in ordinary English. The translation into symbols is

$\sin(x + y) = \sin x \cos y + \cos x \sin y.$

Read that sentence aloud. It should sound exactly as it does when written in plain text. Ultimately, the mathematical symbols are just a different way to spell the same old words.

Here is an example that is *not* an equation:

$$\boxed{x^2 + 2x + 1.}$$

You can tell that it's not an equation because there's no "=". More importantly, it is not even a sentence. When you read it aloud, you find that there is no verb. The reader cannot evaluate whether the writer is correct, because the writer isn't making any claim at all. This math is *worse than wrong.*

In this course, you are expected to write all of your homework and exams in complete sentences. If a homework problem asks you to multiply out $(x+1)^2$, do not just hand in "$x^2 + 2x + 1$." as above. Instead, make a complete statement:

$$\boxed{(x+1)^2 = x^2 + 2x + 1.}$$

Finally, inequalities (which involve "$<$", "$\leq$", "$>$", or "$\geq$" instead of "=") are also statements, just as equations are. Read this one aloud, to convince yourself that it is a sensible English sentence:

$$\boxed{\sqrt{x+1} \geq \sqrt{x}.}$$

A primary goal of this course is to communicate math in succinct, precise prose. Understanding that equations and inequalities are English statements is an important first step.

Exercises

Read each expression aloud. Is it a sentence? If not, why not? If so, then is it true?

A. $\log \sqrt{x^3} - 1$.

B. $(t+3)^2 = t^2 + 9$

Read each statement aloud. Then convert it from symbols to plain text (keeping digits, x, and f, but no other symbols).

C. $\sqrt{101} \geq \sin 0 + 15/1.5$.

D. $f(x) = 3x - 2^x$.

Read each statement aloud. Then convert it from plain text to symbols. (There's some ambiguity, which is the main disadvantage of writing math in plain text. Resolve the ambiguity in such a way that the statement is true.)

E. 3 times the difference of 17 and 9 equals 24.

F. The square root of the quantity x squared, plus 2 times x, plus 1, equals plus or minus the quantity x plus 1.