A. Problem 7.4. (This is related to class last Friday.)

B. Problem 7.14. (This is related to class last Friday.)

Is NP closed under complementation? Nobody knows, but the common suspicion is that NP is not closed under complementation. Explain what is wrong in each of the following "proofs" that NP is closed under complementation. (The proofs are extremely similar, but they make very different mistakes.)

C. Let $A \in NP$. Then there exists an NTM N and natural number k such that $L(N) = A$ and the running time of N is $\mathcal{O}(n^k)$. Define a TM M that, on input w, runs N on w and outputs the opposite of what N outputs. Then $L(M) = \overline{L(N)} = \bar{A}$, and the running time of M is $\mathcal{O}(n^k)$. So $\bar{A} \in NP$.

D. Let $A \in NP$. Then there exists an NTM N and natural number k such that $L(N) = A$ and the running time of N is $\mathcal{O}(n^k)$. Define an NTM M that, on input w, runs N on w and outputs the opposite of what N outputs. Then $L(M) = \overline{L(N)} = \bar{A}$, and the running time of M is $\mathcal{O}(n^k)$. So $\bar{A} \in NP$.