This assignment is due on paper at the start of class on Day 28. Submit polished solutions, including all necessary work and no unnecessary work, in the order assigned.

A. Let $\vec{v}$ be a 3D vector field. Prove that

$$\frac{1}{2}\nabla|\vec{v}|^2 = \vec{v} \times (\nabla \times \vec{v}) + (\vec{v} \cdot \nabla)\vec{v}.$$

(This identity is often used in fluid dynamics, to simplify certain expressions.)

B. Use the identity from problem A to rewrite Euler's equation as

$$\frac{\partial \vec{v}}{\partial t} + (\nabla \times \vec{v}) \times \vec{v} = -\nabla H,$$

where H is what function, exactly?

C. Show that if $\vec{c}(s)$ is a trajectory through the flow, parametrized by arc length s, then $\partial H/\partial s$ satisfies

$$\frac{\partial \vec{v}}{\partial t} \cdot \vec{v} = -|\vec{v}|\frac{\partial H}{\partial s}.$$

(Therefore, if the flow is steady, meaning $\partial \vec{v}/\partial t = \vec{0}$, then H is constant along trajectories. Physicists are always interested in conserved quantities like this.)