This assignment is in two parts. The first part is due at the start of class on Day 18. It will not be collected, but you are expected to complete these exercises, just to practice basic skills. If you feel that you need more practice, then do more problems or talk to me.

16.1 Exercises 13-16, 21, 25, 33, 34

The second part is due on paper at the start of class on Day 19. Submit polished solutions, including all necessary work and no unnecessary work, in the order assigned.

A. In Mathematica, play with the parameters a, b, c, d of the Lotka-Volterra model. As you change these parameters, the shapes of the trajectories change. How can you make the x-population crash to zero (rather than loop back up to a high value on every trajectory)? How can you make the y-population crash to zero?

B. Let X be the set of points in the plane other than the origin:

$$X = \{(x, y) : x \neq 0 \text{ or } y \neq 0\}.$$

Let $\vec{F} = \langle -y/(x^2 + y^2), x/(x^2 + y^2)\rangle$. Notice that $\vec{F}$ is defined on all of X. As we mentioned in class, this vector field is not conservative on X, even though $\frac{d}{dy}F_1 = \frac{d}{dx}F_2$ everywhere on X. Try to find a potential function for $\vec{F}$. Although it is not immediately obvious, you can succeed on the domain $x > 0$. (Hint: You may find the trigonometric identity $\tan(\theta + \pi/2) = -1/\tan(\theta)$ helpful.) You can also succeed on $y > 0$, on $x < 0$, and on $y < 0$. But you cannot succeed on the domain X all at once. Discuss thoroughly.