We'll work on these problems in class on Day 06 of the course. I'll let you know which ones should be handed in.

A. Remember the quadratic formula for solving equations of the form $ax^2 + bx + c = 0$?

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

The quadratic formula is actually a proposition, of the form "If $ax^2 + bx + c = 0$, then...", with some quantifiers at the start too. Write out the complete proposition, including the quantifiers. Use only symbols (not plain English words spelled out), but do not use the "$\pm$" symbol. Be sure to handle any special cases correctly, so that your proposition is true.

B. Prove that the sum of a rational number and an irrational number must be irrational.

C. Recall that the factorial $n!$ of a natural number n is defined by $0! = 1$ and $n! = n(n-1)!$ for all $n \geq 1$. Prove that for all $n \geq 1$,

$$\sum_{k=1}^{n} k k! = (n+1)! - 1.$$

D. Which one is bigger, for large values of n: 2^n or n^2? How large must n be? Prove it.

E. The principle of mathematical induction lets us prove a proposition about natural numbers of the form $\forall n\ P(n)$ by instead proving $P(0)$ and $\forall n\ P(n) \Rightarrow P(n+1)$. A variant of this principle, called *strong mathematical induction*, lets us prove $\forall n\ P(n)$ by instead proving $P(0)$ and $\forall n\ (\forall m \leq n\ P(m)) \Rightarrow P(n+1)$.

1. Using strong induction starting at $n = 2$ (rather than $n = 0$), prove that every natural number $n \geq 2$ can be factored into a product of primes. (This is one half of the fundamental theorem of arithmetic. The other half is about uniqueness; don't prove that.)

2. Explain why strong induction is a valid proof technique.

3. Many facts can be proved by either regular or strong induction. Do you expect that a proof by strong induction is typically easier or harder than a proof by regular induction?

F. Prove the surprisingly important *triangle inequality*: For all real numbers x and y,

$$|x + y| \leq |x| + |y|.$$