These nine problems come from various sources, including your textbook. If you get stuck, then you might try some earlier problems to get warmed up (but do not hand those in).

A. Write each of these statements in the form "if p, then q".

1. Elvis gets sick whenever he rides on a rollercoaster.
2. To win the lottery, it is necessary to buy a ticket.
3. You can run the game *Crysis of Nukemcraft Online* only if you have 8 TB of RAM.

B. Is the following proposition satisfiable? Explain.

$$(p \vee q \vee \neg r) \wedge (p \vee \neg q \vee \neg s) \wedge (p \vee \neg r \vee \neg s) \wedge (\neg p \vee \neg q \vee \neg s) \wedge (p \vee q \vee \neg s)$$

C. Rewrite each of the following compound propositions so that each variable occurs at most once. Prove that your proposition is equivalent to the original using a truth table.

1. $\neg p \wedge (p \vee q)$
2. $p \to (p \wedge q \wedge r)$
3. $((p \wedge q) \to (p \vee q)) \to (q \to r)$

D. A compound proposition is in *disjunctive normal form* (DNF) if it is the disjunction ($\vee$) of zero or more clauses, and each clause is the conjunction ($\wedge$) of one or more propositional variables (p, q, etc.). For example, $(p \wedge q) \vee (s \wedge p \wedge r)$ is in DNF. (There is a symmetric idea of *conjunctive normal form*, of which the proposition in Problem B is an example.) In this problem we learn that any compound proposition is logically equivalent to one in DNF.

1. Explain how to construct, for any particular line of a truth table, a clause that is true for that particular line and false for all other lines of the table.
2. Explain how any proposition is logically equivalent to the disjunction of clauses of the type that you have just constructed.
3. Illustrate your process by giving a DNF proposition equivalent to $(p \wedge (q \vee r)) \to (p \vee \neg r)$.
4. In the opening paragraph of this problem occurs the word *zero* — where you might have expected *one* instead. What is the truth value of the disjunction of zero clauses, and why is it important here? (Hint: What is the sum of zero numbers? What is the product of zero numbers?)

Also do these five exercises from the DLN book:

- 3.19
- 3.20
- 3.25
- 3.55B
- 3.78