A. For any natural number k, let

$$CLIQUE_k = \{\langle G \rangle : G \text{ is an undirected graph that contains a } k\text{-clique}\}.$$

Show that $CLIQUE_k \in P$. (By the way, the $k = 3$ case is Problem 7.9 in our textbook.)

B. In class we've discussed how $CLIQUE \in NP$, and we have almost learned that $CLIQUE$ is NP-complete. How is it possible that $CLIQUE_k \in P$ for any k and yet $CLIQUE$ is NP-complete?

At one point in our proof of Theorem 7.20, we assume the existence of a polynomial-time verifier V, and then conclude that V runs in time n^k for some k. Really we should say that V runs in time $\mathcal{O}(n^k)$ for some k, right? Intuitively, the difference between n^k and $\mathcal{O}(n^k)$ is the constants c and N used in the definition of $\mathcal{O}$. By the way, our proof of the Cook-Levin theorem is also sloppy in this way.

C. Why do we do this? That is, why is it essential to the proof of Theorem 7.20 that the time be (bounded by) n^k?

D. How can we do this? Given that V is $\mathcal{O}(n^k)$-time for some k, how can we conclude that V is n^k-time for some k? Fill in the details. (My answer is moderately long, and goes into detail about c and N.)