There are five problems on this exam.

You have 150 minutes. Notes, calculator, computer, etc. are not allowed.

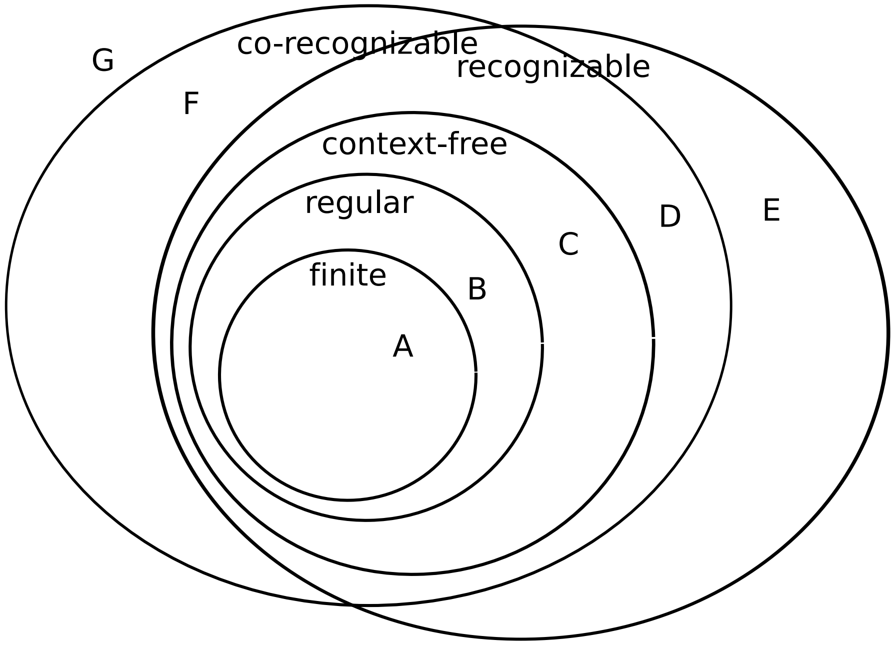
Your solutions should be rigorous and self-explanatory. A typical classmate should be able to understand your solutions. If you are unsure about the level of detail required, then err on the side of giving too much detail. Except where otherwise noted (particularly Problem D), you may cite results proved in class, in the assigned textbook sections, in the homework, and in the earlier exams.

Feel free to ask for clarification. You should certainly ask for clarification if you feel that a problem is misstated.

Good luck.

A. Let $A = \{\langle M, G\rangle : M$ is a Turing machine, G is a context-free grammar, and $L(M) = L(G)\}$. Prove that A is undecidable.

B. The Venn diagram below shows containment relationships among various classes of languages. Within the diagram, there are seven languages labeled $A, B, C, \ldots, G$. For example, A is finite, and B is regular but not finite. For each, give a language over $\Sigma = \{0, 1\}$ that could occupy that position in the diagram. Remember to justify novel answers. B is done for you.



A.

B. $\{0^n 1^m : n, m \geq 0\}$.

C.

D.

E.

F.

G.

C. Prove that TQBF is NP-complete, by reducing SAT to TQBF. (This problem is intended to test whether you understand what a reduction is, and what SAT and TQBF are. So be sure to carry out all details correctly.)

D. A *linear bounded automaton* is a deterministic Turing machine that is not allowed to use any tape beyond its input. (It can, however, sense the left and right ends of the input. Assume that the input arrives bracketed by left and right markers $\vdash$ and $\dashv$, that the tape head cannot overwrite these markers, and that it cannot move beyond these markers.) Working from first principles, derive a bound on the time required by a linear bounded automaton to accept or reject an input of size n. (We've done an example like this in class. You may not cite it.)

E. Prove that $A = \{\langle R, S \rangle : R \text{ and } S \text{ are regular expressions, and } L(R) = L(S)\}$ is in PSPACE.