Carleton College Math 111, Winter 2008, Exam 1

You have 60 minutes.

You may not use any notes or calculator.

Always show your work and explain all of your answers. Good work often earns partial credit. A correct answer with no explanation often earns little or no credit.

If you have no idea how to solve a problem, or if you have forgotten a key formula that you think you need to know, you may ask me for a hint. The hint will cost you some points (to be decided unilaterally by me as I grade your paper), but it may help you earn more points overall.

Good luck.

1. Differentiate the following functions. Do NOT simplify.

A. $y = 5x^6 - 2x^3 + 3x + 1$.

B. $s(t) = \dfrac{t}{\cos t}$.

C. $p = 4^r \cdot (3r - 1)$.

D. $y = \left(2e^{5t} + \sin\left(\sqrt{t}\right)\right)^7$.

2. The Furtwängler Glacier on Mount Kilimanjaro is a giant sheet of ice. Suppose, for simplicity, that it is rectangular of length 600 m and width 100 m. Due to warming climate, its length and width are both shrinking by 5 m per year. Its thickness is a constant 6 m. How fast is the volume of the glacier changing? Simplify your answer, and include units.

3. Suppose that $x = f(t) = \cos(2t) + \sin(3t)$ is the position of a particle at time t. Compute the particle's acceleration. Simplify and clearly mark your answer.

4. Recall that Newton's law of cooling is expressed by the differential equation $\frac{dy}{dt} = k(A - y)$.

A. Explain in words the physical meaning of these five quantities:

t

y

A

k

$\frac{dy}{dt}$

B. We used $k = 0.9$ in the Santa-cooling problem in class. If we had used a lesser (but still positive) value of k, such as 0.5, would Santa cool more slowly or more rapidly? Explain.

C. The value $k = 0.9$ was given to us by the police but not explained further. In words, describe some circumstances of Santa's death that might influence the police estimate for k.

5. Show that $\lim_{h\to 0} h \sin\left(\frac{1}{h}\right) = 0$.

6. Compute the derivative, at $x = 0$, of $y = f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right) & \text{if } x \neq 0, \\ 0 & \text{if } x = 0. \end{cases}$

7. The table below shows the number s of web sites in existence, for each year t in a 10-year period. I wish to model s as a function of t. When I plot the data on a semilog plot, I get a line of slope 1.43 and intercept 2.31. What then is the function $s(t)$? Simplify your answer.

year t	1	2	3	4	5	6	7	8	9	10
# sites s	42	176	735	3070	12800	53600	224000	936000	3910000	16400000

8. Graph $y = e^{2x} - 4$ as precisely as you can (including the correct scale, intercepts, etc.).