These are some sample questions intended to help you prepare for our first exam. I do not promise that questions like these will be on the exam or that the exam questions will all be like these. I do not promise that the exam will be this long/short or difficult/easy. All that you should infer from these sample questions is that the exam will be some mixture of easy, moderate, and difficult problems, that it will include a number of story problems, and that most of the problems are short-answer (as opposed to multiple-choice, for example). You are responsible for all material discussed in class and for all of the sections mentioned in the class schedule, whether or not we've discussed every part of those sections in class.

Remember that calculators and computers will not be used on the exam; therefore you'll have to leave some answers unevaluated, such as $\log 0.92$. Otherwise, you should do as much simplification as possible, as time permits. Show your work in a clear manner, so that another student reading your solution could understand it.

If you are interested in my general tips for studying, check out the little essay *How To Study Calculus* on the class web site.

0. If a particle's position s (in meters) is given as a function of time t (in seconds) by $s = \cos(2t + 1)$, then what is the particle's velocity?

1. Differentiate $y = (x^9 - 2x + 1)\log(2^{\sin x})$.

2. Differentiate $y = f(x) = x^2 - 5$ using only the definition of the derivative.

3. Here are some data about how the voltage V in a certain electronic circuit varies as time t passes. That is, they are data about some function $V = f(t)$. When I plot them on a semilog plot, they seem to lie along a line with slope -0.461 and intercept 3.05.

t	0	1	2	3	4	5	6	7
V	21.6	13.2	8.64	5.28	3.24	1.98	1.29	0.91

A. Exactly which function $V = f(t)$ would you use to fit the data?

B. Based on your model from Part A, when do you expect the voltage to be less than 0.1?

C. How fast is the voltage changing at that time?

4. You are a railroad designer trying to connect two train tracks near an isolated town in Kazakhstan. To describe these two tracks, imagine a coordinate grid placed over the region, with the positive x-axis pointing east and the positive y-axis pointing north. (The units are kilometers.) The track coming in from the next town to the west ends at $(0, 2)$, pointing exactly northeast. The track coming in from the next town to the east ends at $(1, 3)$, pointing exactly west. Find a cubic function $y = f(x) = ax^3 + bx^2 + cx + d$ whose graph joins the two existing tracks with no sharp corners at the joints.

5. Under normal operation, the air inside Capt. Willoughby McWillowsalot's submarine is kept at a comfortable $20°C$. One day, while the submarine is maneuvering deep in enemy waters, the heating system fails. The water outside the submarine is a constant $2°C$, and the submarine starts to cool according to Newton's Law of Cooling. After one day without heating, the temperature inside the submarine is $15.5°$. The captain and his crew will begin to succumb to hypothermia (and die) when the temperature reaches $6°C$. How many days without heat can they go?

6. As accurately as you can (showing intercepts and aymptotes, for example), sketch the graph of

$$y = \frac{x^2 + 5x + 6}{x^3 + x^2 - 2x}.$$

7. Prove the product rule from the definition of the derivative.