Carleton College Math 111, Winter 2008, Exam 2

You have 60 minutes.

You may not use any notes or calculator.

Show your work and explain your answers. Good work often earns partial credit. A correct answer with no explanation often earns little or no credit.

If you have no idea how to solve a problem, or if you have forgotten a key formula that you think you need to know, you may ask me for a hint. The hint will cost you some points (to be decided unilaterally by me as I grade your paper), but it may help you earn more points overall.

Good luck.

1. Compute.

A. $\int t^{17}\, dt$

B. $\int x \sin(3x^2)\, dx$

C. $\int \frac{e^{\sqrt{u}}}{\sqrt{u}}\, du$

2. Differentiate these functions.

A. $s(t) = \ln(\tan(t^2))$

B. $f(x) = x^{(e^{2x}-1)}$

3. After a lot of practice, I've become good at estimating e^a for any number a. But still I have trouble computing $\ln a$. Describe in detail a procedure, based on Newton's method, that I can use to estimate $\ln a$ for any positive number a.

4. You are trying to persuade the government of British Columbia to purchase a $10,000$ km^2 patch of land from the various people who currently own parts of it, to set it aside as a wildlife habitat. The more of it the government purchases, the better, because wild animals prefer large tracts of land far away from human influence; your research suggests that a preserve of x km^2 could sustain $m(x) = \frac{1}{100}x^2$ large mammals. Unfortunately, the more land the government purchases, the more expensive each additional km^2 becomes; it seems that the cost of purchasing x km^2 will be about $c(x) = \frac{25}{10{,}000-x}$ (in thousands of Canadian dollars). How much land should the government buy, to maximize *the number of large mammals protected per dollar*?

5. Let $B = \int_{-1}^{7} \sin(t^2)\, dt$.

A. What kind of object is B (e.g. a number, a function, a region in the plane, a point)? What is its geometric meaning?

B. Express B as a limit of Riemann sums. Be as detailed as possible.

C. Explain how you would find an approximate value for B, if you were given a calculator.

6. In as much detail as possible, graph $y = xe^x$.