Carleton College Math 111, Spring 2008, Exam 2

You have 70 minutes.

You may use a calculator. You may use one 8.5×11-inch sheet of paper with notes written on it (on both sides) by you.

Always show your work and explain all of your answers. Good work often earns partial credit. A correct answer with no explanation often earns little or no credit.

Good luck.

1. In each part, compute dx/dt.

A. $x = \log_7\left(\tan\left(3^t\right)\right)$.

B. $x^3 + t^3 = \sin x$.

C. $x = \left(\sqrt{t}\right)^{\ln t}$.

2. Antidifferentiate.

A. $\int 3s^7 - 2\sqrt{s} + 1 \, ds$

B. $\int \frac{y^4}{y^5 + 4} \, dy$

C. $\int \frac{4^{\ln(t^2)}}{t} \, dt$

3. You are lying on your back in cool grass on a warm summer day. In the sky you see a delightful cloud that resembles a fluffy bunny. The cloud is at an altitude of 2 km and traveling 30 km/h horizontally. When it is directly overhead, how fast must your eyes rotate, to keep staring at it? Include units in your answer.

4. Your company makes tiny plastic figurines of toads. If you make and sell x shipments (of one million toads each), then your manufacturing cost per shipment is

$$m(x) = 3 + \frac{1}{x^{1.3}}$$

(millions of dollars), and your sale price per shipment is

$$s(x) = 3 + \frac{2}{x^{1.9}}$$

(millions of dollars). Find a function $p(x)$ that describes the profit (income minus costs) that you earn from x shipments of toads; then maximize that function on an appropriate interval.

5. A. Given a number $a \neq 0$, describe a procedure based on Newton's method that lets you compute $1/a$ without using long division. Simplify if you can.

B. Use Part A to compute $1/7$ with a starting value of $x_1 = 1$. What happens? Why?