Carleton College Math 111, Spring 2008, Exam 1

You have 60 minutes.

You may use a calculator. You may use one 8.5×11-inch sheet of paper with notes written on it (on both sides) by you.

Always show your work and explain all of your answers. Good work often earns partial credit. A correct answer with no explanation often earns little or no credit.

Good luck.

1. Find the equation of the tangent line to $y = 4x^3 - 3x$ at $x = 2$.

2. Find the acceleration of a particle, given that the particle's position is

$$s(t) = 2\sin(kt) - 9.8\sqrt{t} + 3t^2.$$

3. Differentiate the following functions. Do NOT simplify.

A. $f(u) = \left(\dfrac{1}{2}\right)^{\cos u}$.

B. $r = e^{1.3t}\left(t^{1.3} + t\right)$.

4. The table below shows some data. I wish to model y as a function of x. When I plot the data on a log-log plot, I get a line of slope 1.2 and intercept 2.7. What then is the function $y(x)$? Simplify your answer.

x	3	4	5	6	7
y	56	79	103	129	155

5. Although it may have slipped your mind, you recently piloted an oil tanker from Anchorage, Alaska to Acapulco, Mexico. You measured the following speeds over the course of a single 24-hour period. How far did you travel during that period? Give a range of possible distances.

time	7 AM	10 AM	2 PM	4 PM	5 PM	9 PM	11 PM	7 AM
speed (km/h)	38	42	47	56	52	54	56	44

6. Find a function whose graph might be the one pictured below.
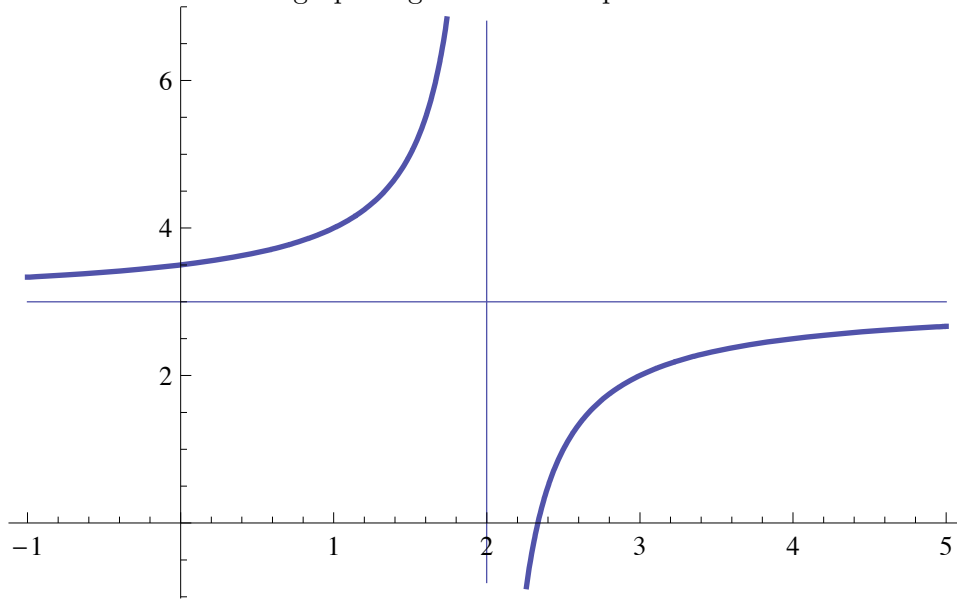
7. Consider the differential equation $\dfrac{dp}{dt} = mp$, where m is a constant.

A. What practical situations does this equation describe?

B. What are the solutions of this equation? Convince me that your answer is correct.

8. From the definition of the derivative, compute $\dfrac{d}{dx}\cos(5x)$. Show all steps.

9. You are given four points (x_0, y_0), (x_1, y_1), (x_2, y_2), and (x_3, y_3). You'd like to find a cubic curve $y = ax^3 + bx^2 + cx + d$ that is tangent at (x_0, y_0) to the line through (x_0, y_0) and (x_1, y_1), and tangent at (x_3, y_3) to the line through (x_2, y_2) and (x_3, y_3). (This problem is very common in computer graphics; for example, it arises when drawing *the letters you are reading right now*.)

    A. Write a system of equations that can be used to find the cubic. Do not solve them.

    B. Find the cubic for $(x_0, y_0) = (0, 0)$, $(x_1, y_1) = (1, 0)$, $(x_2, y_2) = (2, 1)$, $(x_3, y_3) = (3, 1)$.