Name:

I have adhered to the Duke Community Standard.

Signature:

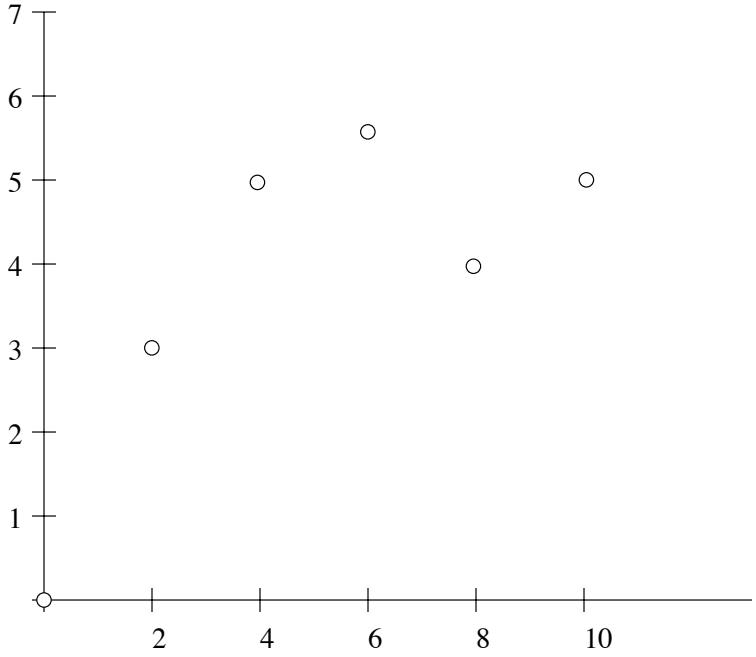
Math 31L-02 Spring 2007 Exam 1

Instructions: You have 70 minutes. You may use your TI-83 or equivalent calculator. Always show all of your work. Partial credit is often awarded. Pictures are often helpful. Give simplified answers, as exact as possible. Put a box around each answer. Ask questions if any problem is unclear. Good luck.

1. Use the definition of the derivative to compute the derivative of the function $f(x) = \dfrac{1}{x+4}$.

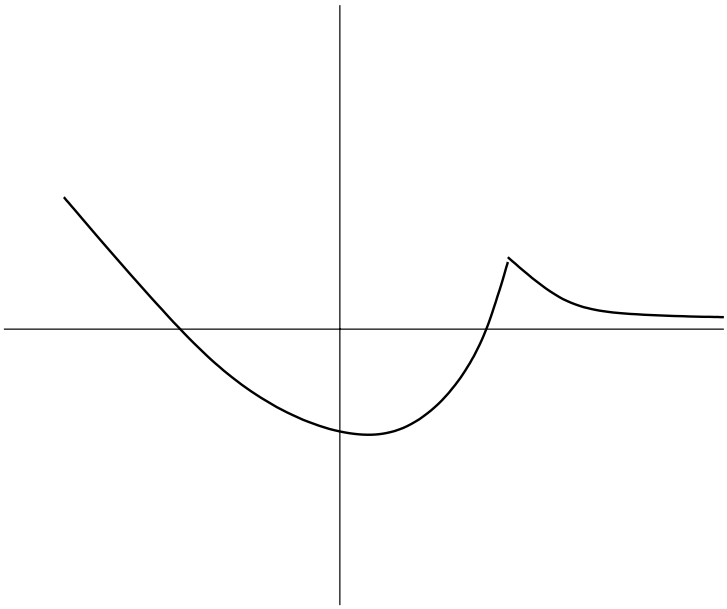
2. Several values are plotted for a function $y = f(x)$ below.
A. Numerically estimate the derivative of f at $x = 0, 2, 4, 6, 8, 10$. Wherever possible, your estimation should average the slopes from the left and right. Enter your answers in the spaces provided.
$f'(0) =$ _____, $f'(2) =$ _____, $f'(4) =$ _____,

$f'(6) =$ _____, $f'(8) =$ _____, $f'(10) =$ _____.



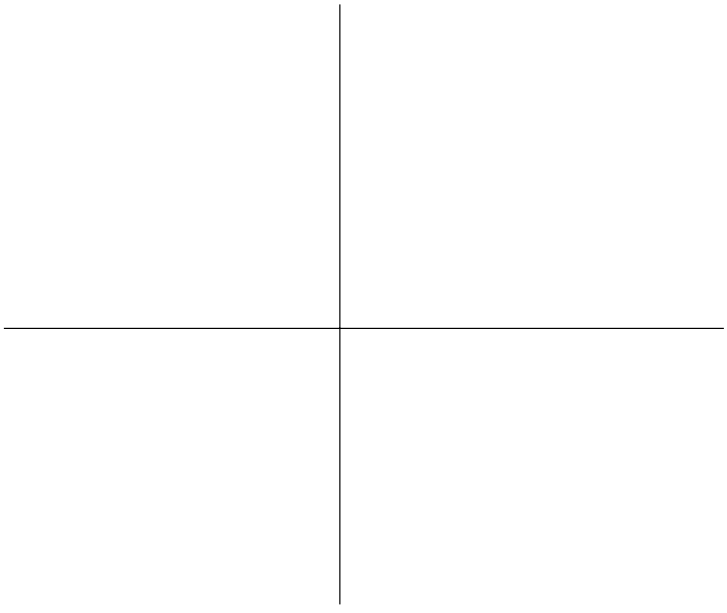
B. Beginning with $f(0) = 0$, reconstruct $f(x)$ from the six derivatives you computed in Part A, using Euler's method with step size $\Delta x = 2$. This reconstructed f will not agree with the one above. Plot it on the same graph above, so that the discrepancy is clear.

3. The graph of a function $y = f(x)$ is shown below. Sketch the graph of $y = f'(x)$ on top of it.



4. Draw a graph $y = f(x)$ such that all three of these conditions are satisfied:

   - $f'(x) < 0$ and $f''(x) < 0$ when $x < 0$,
   - $f'(x) > 0$ and $f''(x) > 0$ when $x > 0$, and
   - f is not differentiable at $x = 0$.



5. Let $f(x) = e^{x^3-3x}$. Your answers to Parts C and D must be explained using algebra, rather than just graphing the function on your calculator.
A. Compute $f'(x)$.

B. Compute $f''(x)$.

C. For which x is $f(x)$ positive?

D. For which x is $f(x)$ increasing?

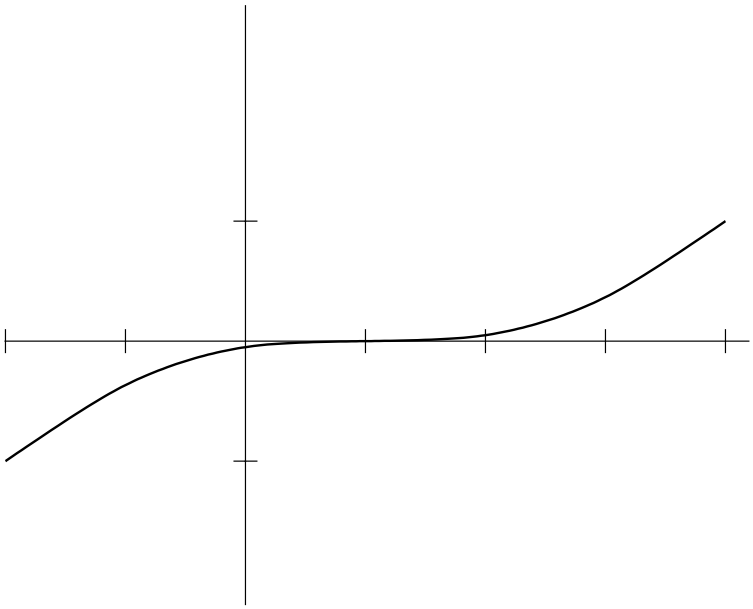
6. A marshmallow is a cylinder of sugar-goo. One day you decide to cook one in a microwave oven. As it heats, it remains a cylinder, but it expands both in radius r and height h. I want to understand how the volume of the marshmallow changes.

A. Find a formula for the derivative of volume with respect to time, t.

B. You observe the following data as the marshmallow cooks. Use the data and the answer to Part A to compute how fast the volume is increasing at time $t = 5$. Include the appropriate units in your answer.

Time t (s)	5	10	15
Radius r (mm)	3	13	24
Height h (mm)	4	9	15

7. Give a function $y = f(x)$ whose graph could be the one below.



8. Compute the following limits, or explain why they do not exist.

A. $\lim_{x \to 0} \sqrt{x+2}\,(1 - e^x)$

B. $\lim_{x \to 2} \dfrac{x^2 - 5x + 6}{x - 2}$

C. $\lim_{x \to \infty} \dfrac{x^2 + 2}{x(x-1)(x+7)}$