Math 32-05/06, Fall 2005, Exam 3

Name:

I have adhered to the Duke Community Standard in completing this examination.

Signature:

Instructions: You have 50 minutes. Calculators are not allowed. Always **show all of your work**. Pictures are often helpful. Partial credit may be awarded. Give **simplified**, **exact** answers, and **draw a box** around them. You may assume the following trigonometric identities:

$\cos^2 x + \sin^2 x = 1$
$1 + \tan^2 x = \sec^2 x$
$\cot^2 x + 1 = \csc^2 x$
$\sin(x + y) = \sin x \cos y + \cos x \sin y$
$\cos(x + y) = \cos x \cos y - \sin x \sin y$
$\sin 2x = 2 \sin x \cos x$
$\cos^2 x = (1 + \cos 2x)/2$
$\sin^2 x = (1 - \cos 2x)/2$
$\cosh^2 x - \sinh^2 x = 1$
$1 - \tanh^2 x = \mathrm{sech}^2 \mathrm{x}$
$\coth^2 x - 1 = \mathrm{csch}^2 \mathrm{x}$
$\sinh(x + y) = \sinh x \cosh y + \cosh x \sinh y$
$\cosh(x + y) = \cosh x \cosh y + \sinh x \sinh y$
$\sinh 2x = 2 \sinh x \cosh x$
$\cosh^2 x = (\cosh 2x + 1)/2$
$\sinh^2 x = (\cosh 2x - 1)/2$

1. Integrate.

A.

$$\int \frac{5x^2 - 16x + 15}{x(x^2 - 4x + 5)}\, dx$$

B.

$$\int x\sqrt{1 - x^2}\, dx$$

C.

$$\int x \ln x - x \, dx$$

2. Let $\Gamma(t) = \int_0^\infty x^{t-1} e^{-x} \, dx$. What is $\Gamma(2)$?

3. For each sequence, find its limit or show that it diverges.

A.

$$a_n = \frac{1 + \cos^2 n}{n}$$

B.

$$a_1 = -3; a_{n+1} = -2 + \frac{3}{a_n}$$

C.

$$a_n = \frac{\ln 2n}{\ln 3n}$$

4. On the last test we studied a falling object with (downward) acceleration

$$\frac{dv}{dt} = 32 - \frac{1}{2}v^2$$

(in ft/s^2), which is the sum of 32 (the acceleration due to gravity) and $-\frac{1}{2}v^2$ (the acceleration due to drag). On this test, we will change the drag term to $4v - \frac{1}{2}v^2$. So now the acceleration is

$$\frac{dv}{dt} = 32 + 4v - \frac{1}{2}v^2.$$

Solve this differential equation for $v(t)$, assuming that the object starts with a velocity of 4 ft/s. You may want to use the fact that the derivative of $\tanh^{-1} x$ is $1/(1 - x^2)$.

5. Does the series $\sum_{n=1}^{\infty} \sqrt[n]{2}$ converge? If so, to what? Explain.

6. In its first year of operation, Jimi's olive oil company earned revenues of 50 million Australian dollars. Since then, things have not gone so well. Yearly revenues have decreased by 10% each year. If this trend continues indefinitely, how much total revenue will the company earn in its history?

7. In general, what does $\sum_{n=0}^{\infty} a_n = A$ mean? That is, how is the value of a series defined?