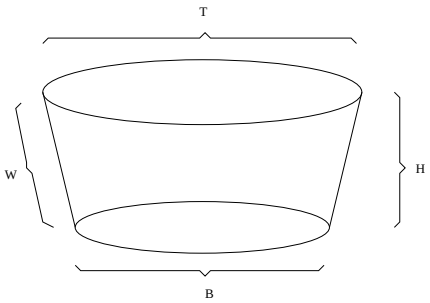
Truncated Cone Problem, by Joshua R. Davis, Math 113, 24 October 2004

This is a problem for extra credit, worth one homework assignment. It is due 22 November 2004. While solving it, you may consult your textbook, you may use your non-graphing calculator, and you may ask me for help. You may not consult your classmates or other people. You may not consult other books, web pages, computer software, superintelligent robots, or other resources. You should be able to solve this problem using the tools we have already studied in this class – in fact, just the Pythagorean theorem, arc length, and sine and cosine.

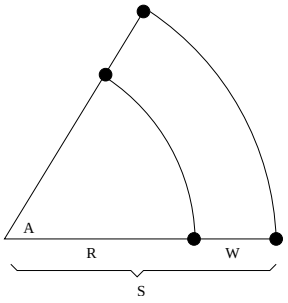
This is a real problem asked me by my father a couple of weeks ago. He wants to plant bamboo in a certain section of his yard. But bamboo spreads agressively, so it is often planted in a "container" sunken in the ground. His container will be a truncated cone:




The container has no top and no bottom – just a side. When bamboo planted inside the container sends out runners, they spread horizontally, running into the side of the cone and then following it up to the surface. Eventually, the bamboo will fill out the container without expanding beyond it.

The diameter of the cone at the top is $T$, and the diameter at the bottom is $B$. The side length is $W$. Note that $W$ is different from the height $H$; $W$ is a length measured along a slant.

The container will be made out of sheet metal. That is, a single piece of sheet metal will be cut, curled up, and welded at its edges to make the container. It's not hard to see that, for the final product to be a truncated cone, the piece of sheet metal has to be cut along circular arcs. It has to look something like the piece in this picture with its four corners marked. Here $W$ is the same $W$ mentioned above.




The first three questions ask for formulae, while the last question asks for approximate numerical values. Write up your answers neatly, showing your work, simplifying your answers as much as possible, and explaining any complicated reasoning you perform and any new notation you introduce.

1. Find a formula for the height $H$ of the truncated cone, in terms of $T$, $B$, and $W$.

2. Find formulae for the angle $A$ (in radians), the inner radius $R$, and the outer radius $S$.

3. Sheet metal normally comes in rectangular pieces, so the curved piece will have to be cut out of a rectangle, with some waste. What is the smallest rectangle, out of which the curved piece could be cut? That is, give me formulae for the length and width of this rectangle.

4. My dad actually specified $T = 3$, $B = 2.5$, and $W = 2.5$ (all in feet). Plug these values into your formulae from questions 1, 2, and 3. Give me numerical values for $H$, $A$, $R$, $S$, and the length and width of the rectangle. (If these numbers seem too large or small, you might go back and check your formulae.)